D. Polynomials (Harman P 160)

Review Harman Pages 160-161.

Let $a_0, a_1, \ldots, a_n$ be $n+1$ arbitrary numbers with $a_n \neq 0$. Then, the function

$$P(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0$$

is a polynomial of degree $n$. The $n+1$ constants $a_0, a_1, \ldots, a_n$ are the coefficients of the polynomial. A polynomial is a real polynomial if all its coefficients are real numbers. This text considers only polynomials with real coefficients unless otherwise stated, because these are associated with mathematical models of physical systems.

The numbers $z$ that are solutions to the equation

$$P(z) = 0$$

are called the roots or sometimes the zeros of the polynomial. The values of the roots are not necessarily real numbers. Thus, a root $z$ may have the form $z = x + iy$, where $i$ is the imaginary number $\sqrt{-1}$. In electrical engineering problems, this is often written $j$ so that no confusion would result with the current if it is designated by $i$. As described in Chapter 2, the number $\bar{z} = x - iy$ is the complex conjugate of $z$. The notation $z^*$ is also used to designate the complex conjugate of $z$. 
Important properties of real polynomials and their roots are as follows:

1. A polynomial of degree $n \geq 1$ has $n$ roots.

2. A polynomial of odd degree has at least one real root.

3. If $z$ is a complex root of a real polynomial, then the complex conjugate $\bar{z}$ is a root also.

The polynomial $P(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0$ can always be written in the form

$$P(z) = (z - z_1)(z - z_2) \cdots (z - z_n) a_n$$  \hspace{1cm} (3)$$

as the product of linear factors using the roots $z_i, i = 1, 2, \ldots, n$ of $P(z) = 0$. 

MATLAB roots and poly

\[
\begin{align*}
\text{>> } \% p &= x^3 - 7x^2 + 40x - 34 \\
\text{>> } r &= \text{roots}([1 -7 40 -34]) \\
r &= \\
3.0000 + 5.0000i \\
3.0000 - 5.0000i \\
1.0000 \\
\text{>> } p &= \text{poly}(r) \\
p &= \\
1.0000 & -7.0000 & 40.0000 & -34.0000
\end{align*}
\]

See also \texttt{conv} and \texttt{deconv} Harman p431 and \texttt{polyval} Harman p350.

\textbf{Polynomial fit (7.1, P351)} Go over Example 7.1 Harman P353.
Handout 4. Functions, Sequences, Series (Harman P 276)
Review Harman Pages 276-277, 282-296.

1. Continuous functions definition P276
2. Sequences Section 6.2 P 282
3. Infinite Series P 284-287
4. Geometric Series P 287
5. Series of functions P 288
6. Power Series P 290

**Taylor Series (6.3, P292)**
After a review of Taylor series, we can visit Euler’s formula again using results in Table 6.3 P295. Consider the Taylor series

\[ e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \cdots \]

Collecting terms using the fact that \( i^2 = -1, \ i^3 = -i, \) and so on yields

\[
e^{i\theta} = \left(1 - \frac{(\theta)^2}{2!} + \frac{(\theta)^4}{4!} - \cdots\right) + i \left(\theta - \frac{(\theta)^3}{3!} + \frac{(\theta)^5}{5!} - \cdots\right)
\]

\[= \cos \theta + i \sin \theta \quad (4)\]

Notice that in all the Taylor series examples, if the argument such as \( \theta \) in the expansions for \( \sin \) and \( \cos \) only a few terms may be needed to satisfy the precision requirements of a problem. For example, if \( \theta \) is 0.05 radians (about 3 degrees),

\[\sin(0.05) = 0.0500\]

to four decimal places. In other words, \( \sin x \approx x \) when \( x \) is small.
EXAMPLE Series Output of a Full Wave Rectifier

When the diodes are reversed biased by the voltage $V_p$ on the capacitor, the voltage across the resistor and capacitor are equal so that

$$i = \frac{v}{R} = C \frac{dv}{dt}$$

The differential equation

$$\frac{dv}{dt} + \frac{v}{CR}$$

with initial conditions $v(0) = V_p$

The solution is $v(t) = V_p e^{\lambda t}$ with $\lambda$ the solution to the characteristic equation $\lambda + 1/RC = 0$ as described in Harman p216. The decay in the voltage from the peak is computed by expanding the exponential and evaluating the first terms at $t = T/2$ to yield

$$V_{min} = V_p \exp\left(\frac{-t}{Rt}\right) \approx V_p \left(1 - \frac{T/2}{RC}\right) = V_p \left(1 - \frac{1}{2fRC}\right) \quad (5)$$

The figures illustrate the problem. The idea is to determine the ripple.
Figure 2.11 The ripple is the undesirable portion of the output. (a) Rectifier output; (b) dc component; (c) ripple component.

Figure 2.12 A filter is used to reduce the ripple.

Figure 2.13 Full-wave bridge rectifier with a capacitor filter.

Figure 2.14 Waveforms for the full-wave rectifier with a capacitor filter.

Figure 2: Caption for 5131FullWaveCogdell0001
The input wave in is a 120 volt rms, 60 Hertz sine wave so that \( v(t) = 120\sqrt{2}\sin(2\pi60t) \). The time constant for the decay is

\[
\tau = RC = 100 \times 2000 \times 10^{-6} = 200 \text{ ms}
\]

which is long compared to the period of the sine wave at \( T = 1/60 = 16.67 \) ms. Using the values in the series yields a value of about 0.04 for the exponent at \( t = T/2 \) which indicates that the approximation to \( V_{min} \) is reasonable.
Filtered full-wave rectifier

Find the ripple and dc voltage out of the filtered full-wave rectifier in Fig. 2.16.

\[
\begin{align*}
L = 125 \Omega \\
I = 1.0 \text{ A}
\end{align*}
\]

Figure 2.16 Full-wave rectifier circuit with a capacitor filter.

Solution:
The input ac to the rectifier has an rms voltage of 120 V at 60 Hz. Thus, the peak value is
\[ V_{\text{rms}} = 120 \text{ V} \]
The peak voltage across the load is
\[ V_{\text{load}} = I \cdot R = 1.0 \text{ A} \cdot 120 \Omega = 120 \text{ V} \]
The time constant is
\[ \tau = RC = 100 \Omega \cdot 200 \mu\text{s} = 200 \mu\text{s} \]
which is long compared to the period of 16.7 ms. Thus the approximate analysis is valid. The maximum voltage across the load is
\[ V_{\text{peak}} = \sqrt{2} V_{\text{rms}} = 170 \text{ V} \]

The minimum voltage is given by Eq. (2.11)
\[ V_{\text{min}} = 170 \left[ 1 - \frac{1}{2 \pi f C} \right] = 163 \text{ V} \]
Thus, the ripple voltage is
\[ V_{\text{ripple}} = \sqrt{2} V_{\text{rms}} - V_{\text{min}} = 17 \text{ V} \]
The dc voltage across the load is the average between the maximum and minimum, 166 V. The dc current in the load is calculated by dividing the load resistance, 120 ohm, by the load voltage, 166 V.

What if? What if the full-wave rectifier is replaced by a half-wave rectifier?

Figure 3: Cogdell FullWave Ex2_14