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\textbf{RELATIONSHIP BETWEEN \(F_S\) AND \(F_T\)}

\% Pulse plot and Pulse Train Equal amplitudes and pulse widths
\% The Pulse train frequencies samples the pulse spectrum
\% Harman pages 389, 401-403; K\&H pages 112 and 119-120
\% 
\texttt{clear, clf}
\texttt{A=1, tau=1, T0=2; \% (Expect X=0 when n= 1,2, ... w*tau=n*2*pi}
\texttt{w=[-5*pi:.005:5*pi]; \% Plot from w=0 to 5*pi rad/sec}
\texttt{w=w+eps}
\texttt{Wpulseft= (A*tau)*sin(w*tau/2)./(w*tau/2);}
\texttt{figure(1)}
\texttt{wzero=zeros(size(w)); \% Put in a zero line and plot f Hz.}
\texttt{plot(w,Wpulseft,w,wzero)}
\texttt{xlabel('omega radians/sec'),ylabel('F(omega)'),grid}
\texttt{title('Fourier Transform of pulse A=1, \tauau=1')}
\% Divide the Fourier Transform of the pulse by T0
\texttt{Wpulse= (1/T0)*(A*tau)*sin(w*tau/2)./(w*tau/2); \%}
\texttt{wzero=zeros(size(w)); \% Put in a zero line and plot f Hz.}
\% Add the pulse train spectrum T=2
\% Form n*w0 for the pulse train w0=2*pi/T0=pi
\texttt{nw0=pi*[-5:1:5];}
\texttt{nw0=nw0+eps}
\texttt{Ftrain= (A*tau/T0)*sin(nw0*tau/2)./(nw0*tau/2); \%At/(n*pi*T0)}
\texttt{figure(2)}
\texttt{plot(w,Wpulse,w,wzero), hold}
\texttt{stem(nw0,Ftrain), xlabel('omega radians/sec')}
\texttt{ylabel('Pulse and pulse train')}
\texttt{title('The pulse train spectrum samples the pulse spectrum/T0')}
\texttt{gtext('A=1, \tauau=1, T0=2 for the pulse train'), gtext('pi'), grid}
\texttt{hold off}

\textbf{RELATIONSHIP TO FOURIER SERIES}

Comparing the coefficients of the Fourier series of Example 8.7 for a periodic pulse train of rectangular pulses and the Fourier transform of Example 8.11 for a single pulse shows that the series coefficients are

\[ a_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t)e^{-j\omega n t} dt = \frac{A_T \sin(n\omega_T T/2)}{n\omega_T T/2} \]

and the transform is

\[ F[f(t)] = F[\omega] = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt = A_T \frac{\sin(\omega T/2)}{\omega T/2} \]

By comparing the two results, it is clear that designating the transform \(F[\omega] = F[f(t)]\),

\[ \frac{F[n\omega_0]}{T} = \frac{A_T \sin(n\omega_T T/2)}{n\omega_T T/2} \]

Thus, we conclude that the Fourier series coefficients are obtained by sampling the Fourier transform at the points \(n\omega_0\) and dividing by the period \(T\). However, the Fourier series itself is a continuous function of time, but the Fourier transform is a function of \(\omega\) in the frequency domain.
The pulse train spectrum samples the pulse spectrum/T0

\[ f(t) = \frac{A \tau}{T_0} + \frac{2A \tau}{T_0} \sum \frac{\sin n \omega_0 \frac{T_2}{2}}{n \omega_0 \frac{T_2}{2}} \cos(n \omega_0 t) \quad \omega_0 = \frac{\pi \tau}{T_0} = \pi \]

\[ A=1, \quad T=1 \]

\[ \sum_{k=-\infty}^{\infty} C_k e^{j k \omega_0 t} \]

\[ C_0 = \frac{1}{2} a_k \]
**FOURIER PROPERTIES**

**TIME SHIFT IN SERIES**

\[ f(t) = \frac{A_T}{T_0} + \sum_{n=1}^{\infty} q_n \cos \left( \frac{2n\pi T}{T_0} t \right) \]

\[ \omega_0 = \frac{2\pi}{T_0} \]

\[ q_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} A \cos \left( \frac{2n\pi}{T_0} t \right) = \frac{2A}{n\pi} \sin \left( \frac{2n\pi T}{2T_0} \right) \]

\[ = \frac{2A_T}{T_0} \left[ \sin \left( \frac{2n\pi T}{2T_0} \right) \right] \]

Thus,

\[ f(t) = \frac{A_T}{T_0} + \frac{2A_T}{T_0} \sum_{n=1}^{\infty} \frac{\sin \left( \frac{n\pi T}{T_0} \right)}{\left( \frac{n\pi}{T_0} \right)} \cos \left( \frac{2n\pi T}{T_0} \right) \]

**SHIFT BY T_0**

\[ f(t) = \frac{A_T}{T_0} + \frac{2A_T}{T_0} \sum_{n=1}^{\infty} \frac{\sin \left( \frac{n\pi T}{T_0} \right)}{\left( \frac{n\pi}{T_0} \right)} \cos \left( \frac{2n\pi (t-T_0)}{T_0} \right) \]

**EVEN WRT (t-T_0)**

\[ f(t) = \frac{A_T}{T_0} + \frac{2A_T}{T_0} \sum_{n=1}^{\infty} \frac{\sin \left( \frac{n\pi T}{T_0} \right)}{\left( \frac{n\pi}{T_0} \right)} \cos \left( \frac{2n\pi (t-T_0)}{T_0} \right) \]
I consider the time shift theorem for series

\[ X(t-t_0) \leftrightarrow e^{-j2\pi t t_0 / T_0} \{C_k\} \]

where \(C_k\) are the complex exponential coefficients

\[ C_k = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-j2\pi k t / T_0} dt \quad k = 0, \pm 1, \ldots \]

Note \( C_k = \frac{1}{2} (a_k - j b_k) \) i.e. \( C_{-k} = \frac{1}{2} (a_k + j b_k) \) \( k = 1, 2, \ldots \)

so \( C_k = \frac{A}{T_0} \sin \left( \frac{n \pi t^2}{2T_0} \right) \times e^{-jK \frac{2\pi t_0}{T_0}} \)

Using (1), the change is

\[ \cos \left( \frac{2\pi t_0}{T_0} \right) \times e^{-jK \frac{2\pi t_0}{T_0}} = \left[ e^{-j\frac{2\pi \omega t}{2}} + e^{j\frac{2\pi \omega t}{2}} \right] e^{-jK \frac{2\pi t_0}{T_0}} \]

This is

\[ \cos \left[ \frac{2\pi t_0}{T_0} (t-t_0) \right] \]

and letting \( K = n \)

we have (2)