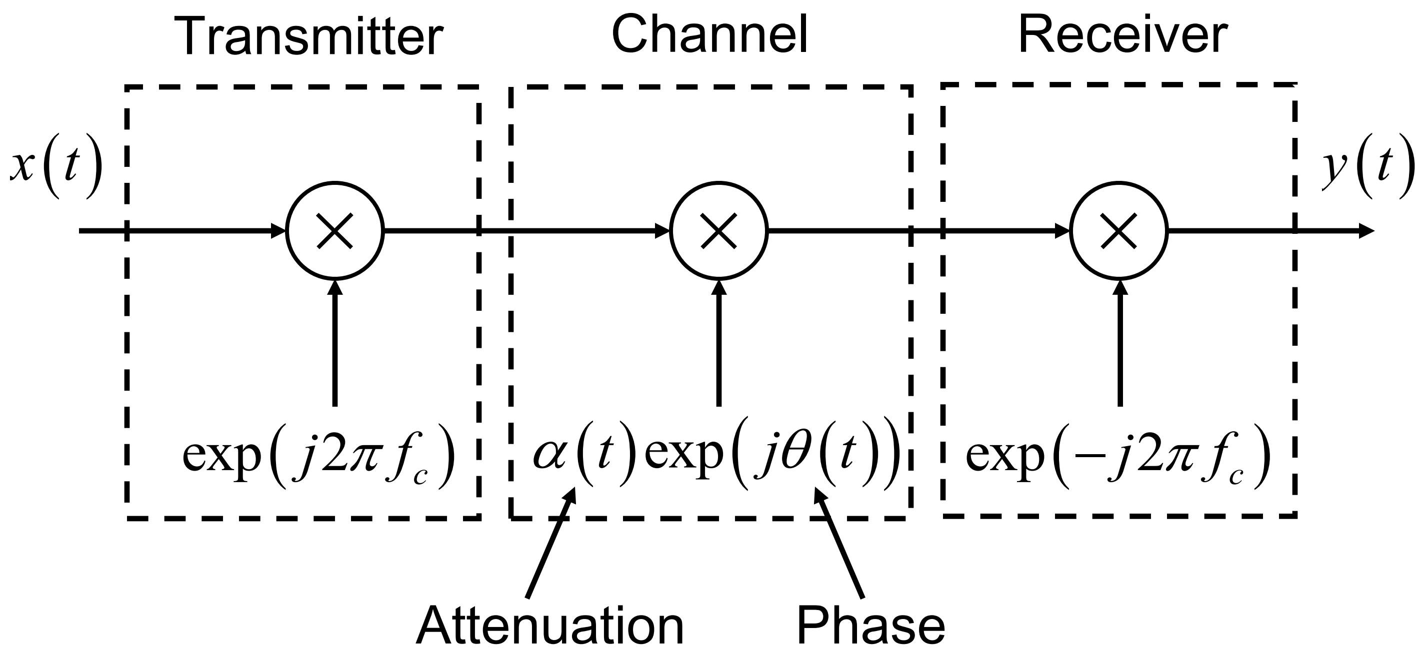
Statistical modeling

```
Multipath propagation on the received
field strength and the temporal
variations (movement) assuming the
transmit signal is a sinusoid.  Valid
for narrow band signals although most
wireless systems today are wideband
(large bandwidth because of high data
rates or multiple access scheme).
```

A narrowband system described in complex notation (noise free)



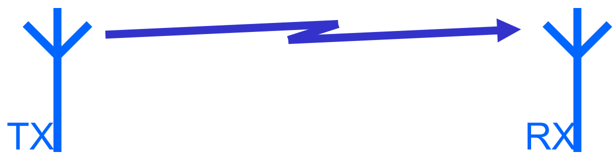
In: $x(t) = A(t)\exp(j\phi(t))$ sinusoidal input assumption good for narrow band channel

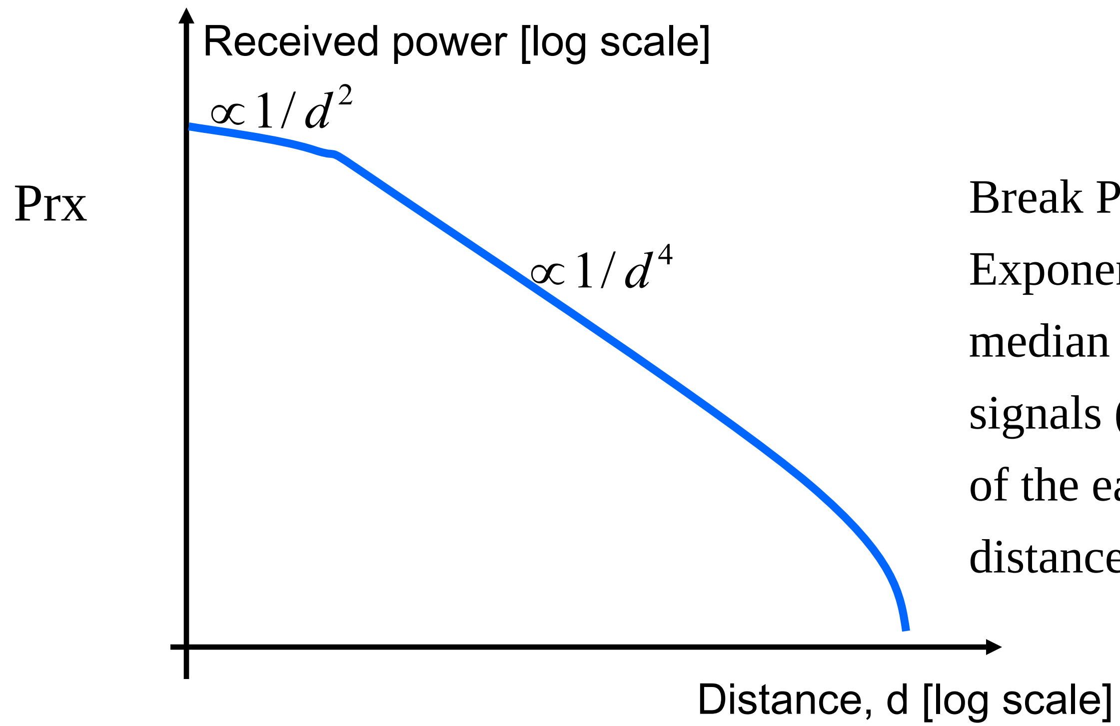
Out: $y(t) = A(t)\exp(j\phi(t))\cancel{\exp(j2\pi f_c t)}\alpha(t)\exp(j\theta(t))\cancel{\exp(-j2\pi f_c t)}$

$= A(t)\alpha(t)\exp(j(\phi(t)+\theta(t)))$ forget delay dispersion, frequency response variations

It is the behavior of the channel attenuation and phase we are going to model. [Channel Gain --> Rayleigh & Rice Distributions]

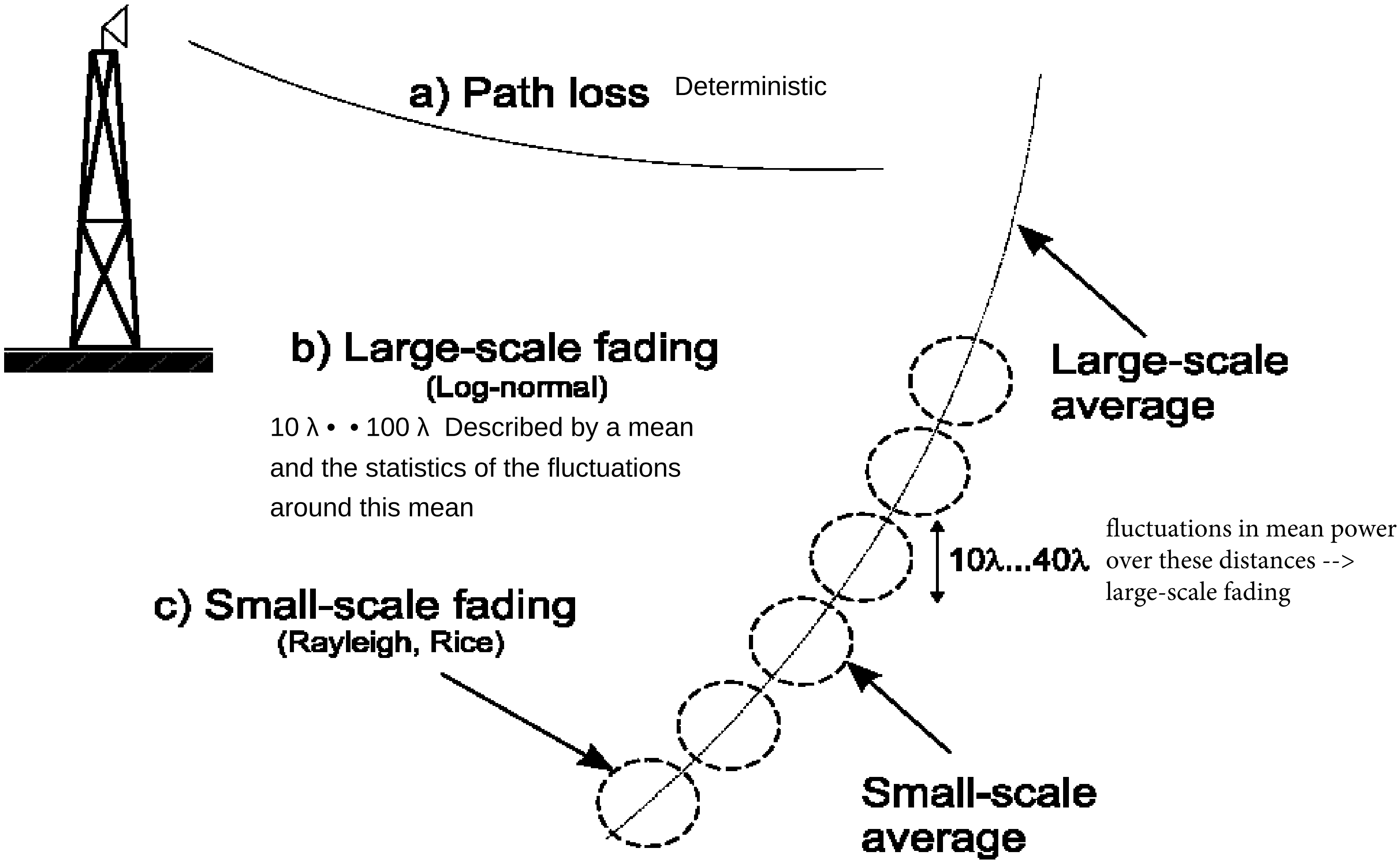
THE RADIO CHANNEL - Path Loss





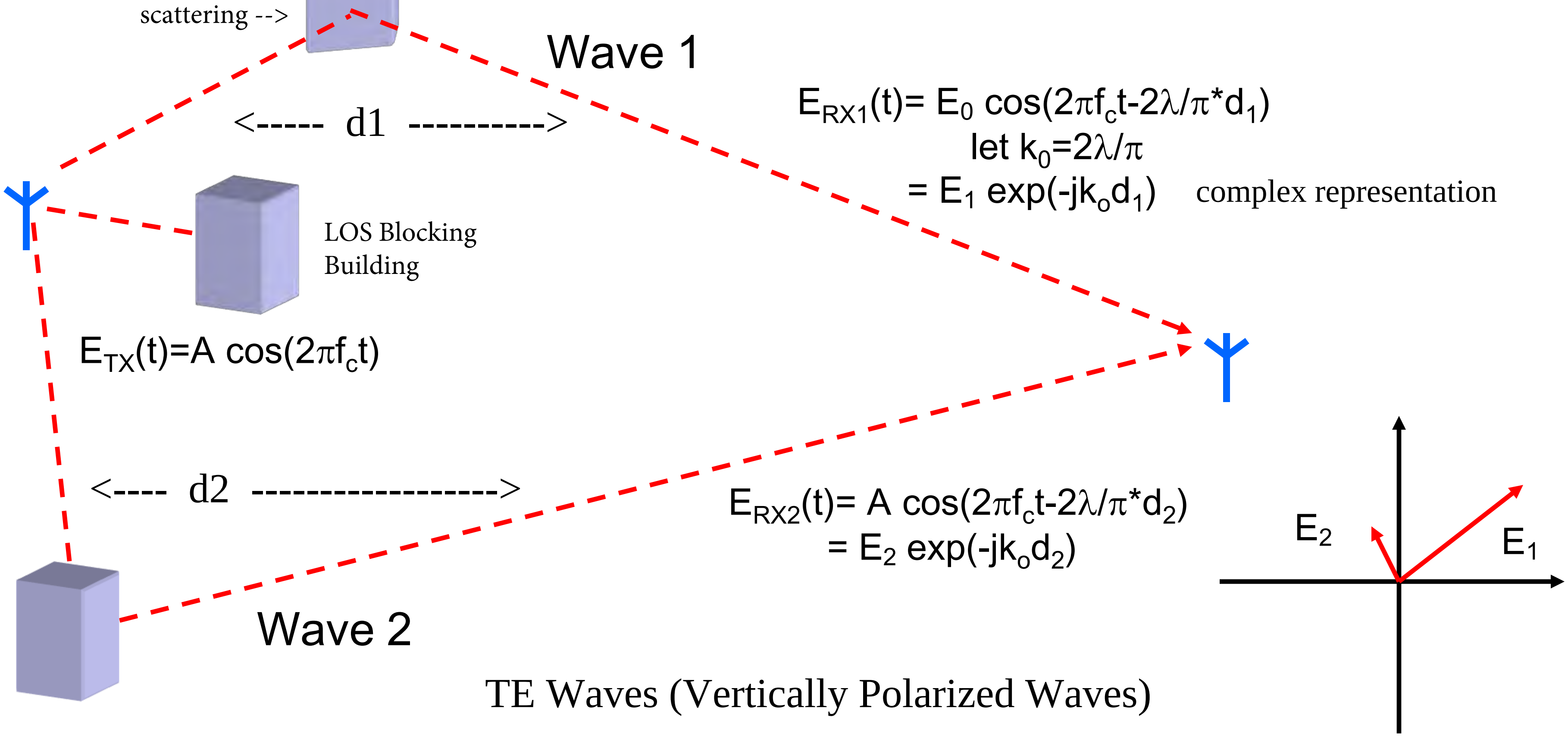
Break Point Model: Exponent of -4 is a good median value. For LOS signals (GHz) curvature of the earth will limit path distance.

What is large scale and small scale?



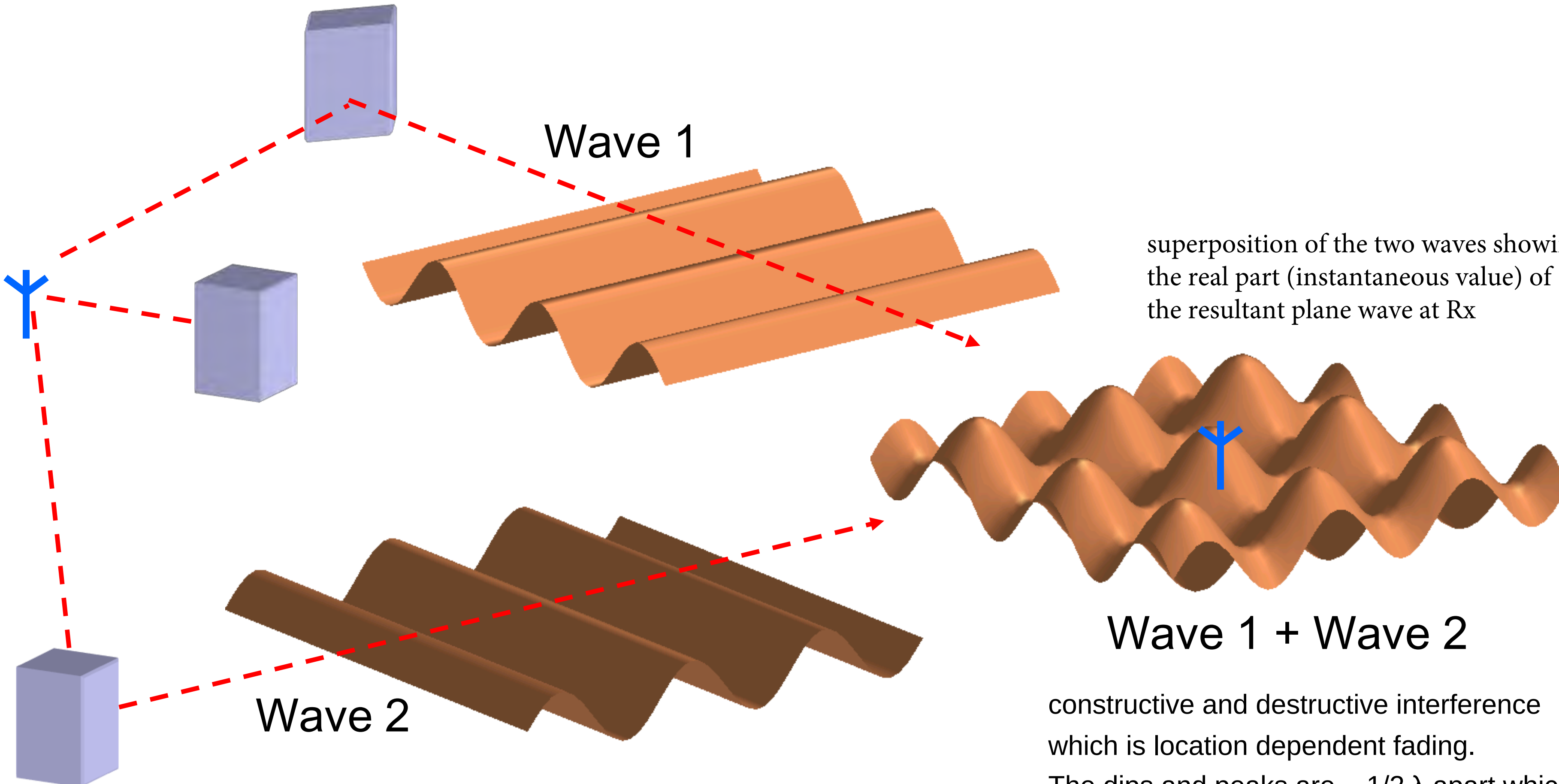
Small-scale Fading for two waves

Two Path Model with no direct LOS path (which is shown being blocked by a very close and big building between the two stations)



TE Waves (Vertically Polarized Waves)

Small-scale Fading of two waves



superposition of the two waves showing the real part (instantaneous value) of the resultant plane wave at Rx

constructive and destructive interference which is location dependent fading.
The dips and peaks are ~ 1/2 λ apart which at 900 MHz is 16 cm

THE RADIO CHANNEL
Small-scale fading or fast fading

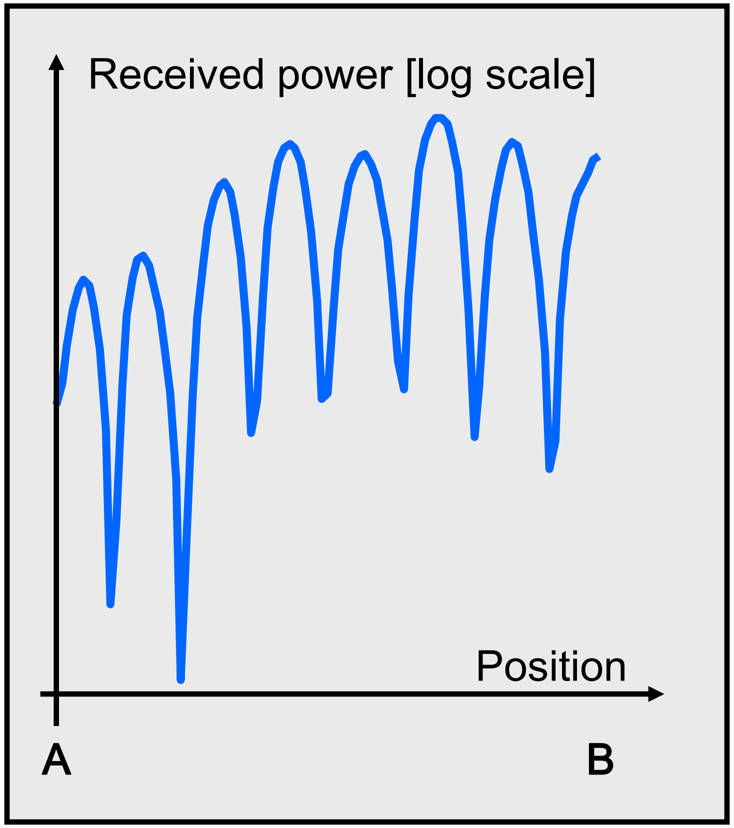
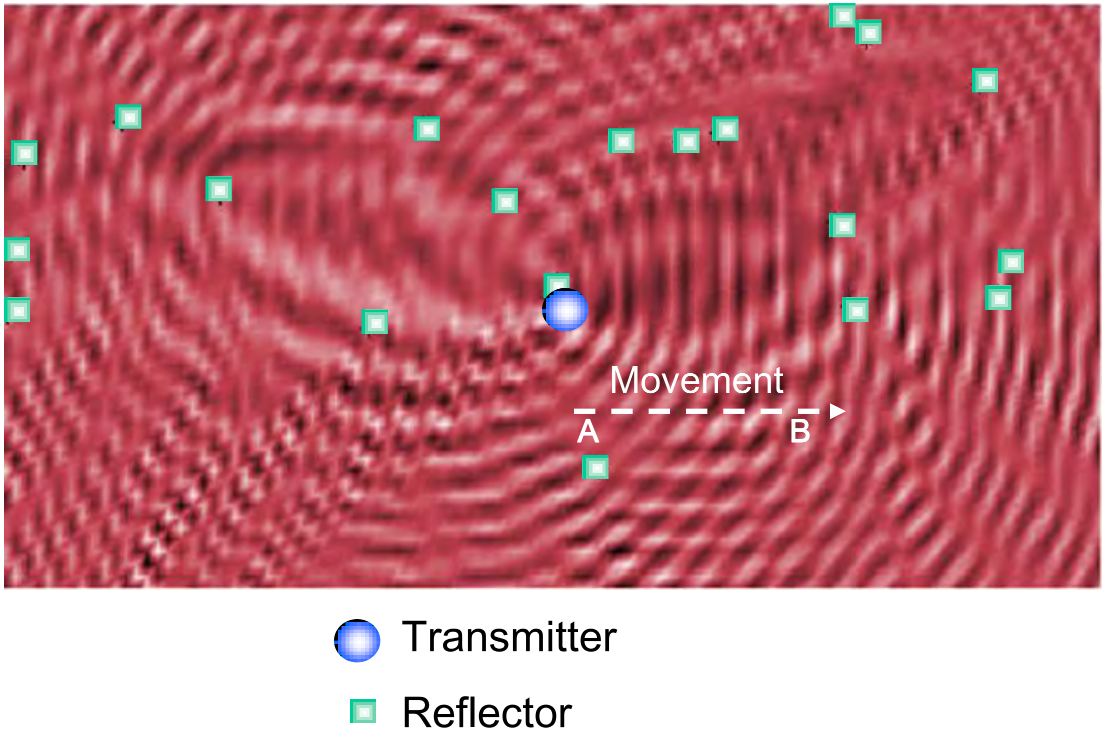


Illustration of interference pattern from above



Re{E} & Im{E} parts are independent and normally distributed (Gaussian Distribution)

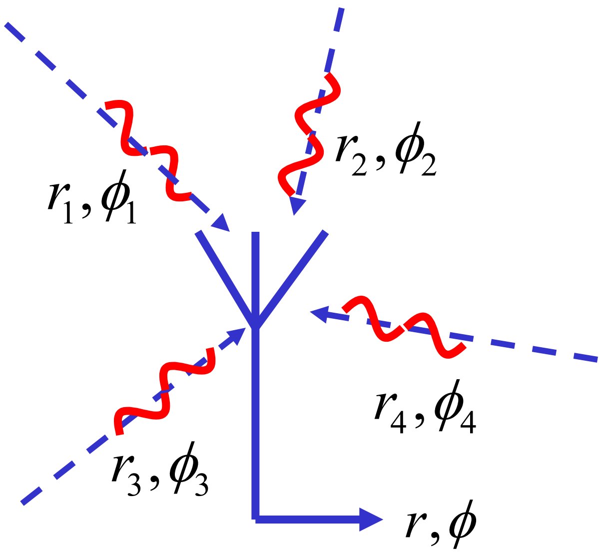
Rayleigh Fading Development - Textbook

The textbook takes two different approaches for developing a description of small-scale fading without a dominant component.

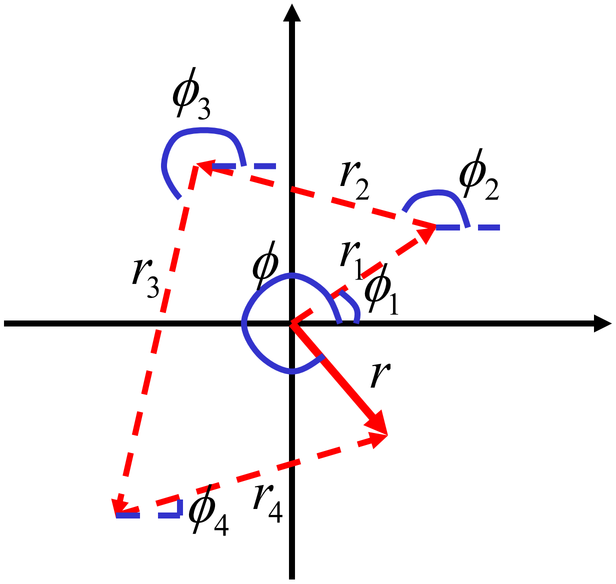
1. Describes a computer experiment in Section 5.4.1 consisting of 8 sinusoidal signals (waves) E each with an amplitude, an angle of incidence from the interfering object (IO) and a phase. The waves are superimposed resulting in a summation of complex field strengths or as pictorially shown in the lecture notes, the vector addition of complex random phasors. The experiment results are analyzed looking at the distribution of the envelope for the received signal - the amplitude and the statistics of the amplitude. The probability density function (pdf) of the amplitude (|E|) matches a Rayleigh distribution (Figure 5.13 on page 78)
2. A mathematical derivation of the Rayleigh distribution is in Section 5.4.2 The rigorous derivation must take into account the Doppler Shift (to be described) of numerous plane waves created by reflection/scattering from different Interfering Objects (IOs). The total field strength E is the sum of many random variables described as both in-phase [I(t)] and quadrature-phase [Q(t)] components. Appendix 5.A shows that the amplitudes of these multipath components fulfill the central limit theorem. The pdf of such a sum (complex) is a Gaussian distribution. Appendix 5.B derives the statistics of the amplitude (r) and phase (Ø) of the received signal showing that the pdf of the phase is a uniform distribution and that the pdf for the amplitude is a Rayleigh distribution.

Small-scale fading
Many incoming waves

Many incoming waves with independent amplitudes and phases



Add them up as phasors



$$r\exp\left(j\phi\right)=r_1\exp\left(j\phi_1\right)+r_2\exp\left(j\phi_2\right)+r_3\exp\left(j\phi_3\right)+r_4\exp\left(j\phi_4\right)$$

$\mathrm{Re}(r) \in N(0,\sigma^2)$

Small-scale fading
Many incoming waves

Re and Im components are sums of many independent equally distributed components

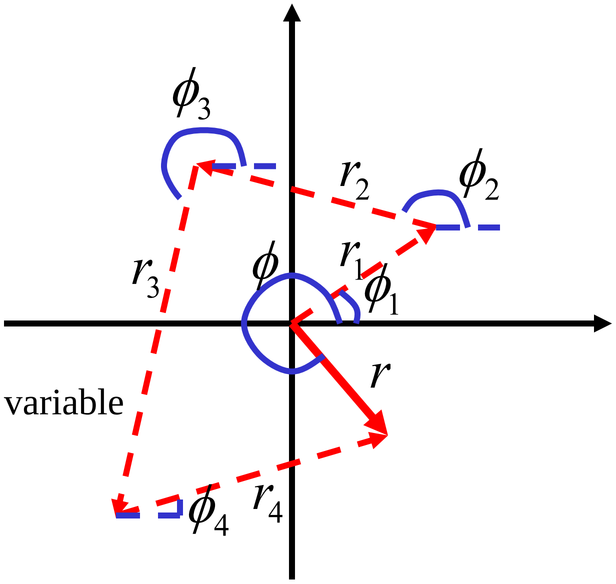
/ variance

$$\mathrm{Re}(r) \in N(0,\sigma^2)$$

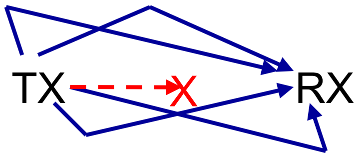
Re(r) and Im(r) are independent

A Rayleigh Distribution describes the magnitude of the complex stochastic variable

The phase of r has a uniform distribution (Figure 5.14)

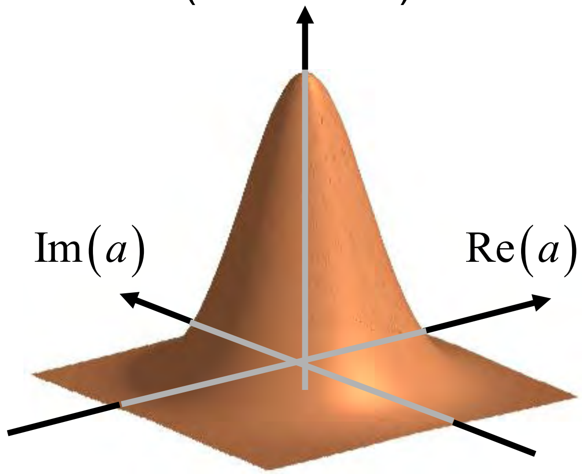


Small-scale fading / Rayleigh fading



No dominant component
(no line-of-sight)

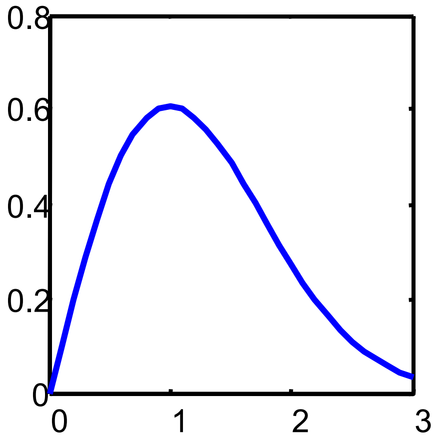
Tap distribution
2D Gaussian
(zero mean)



$r = |a|$

Amplitude distribution (Fig 5.13)

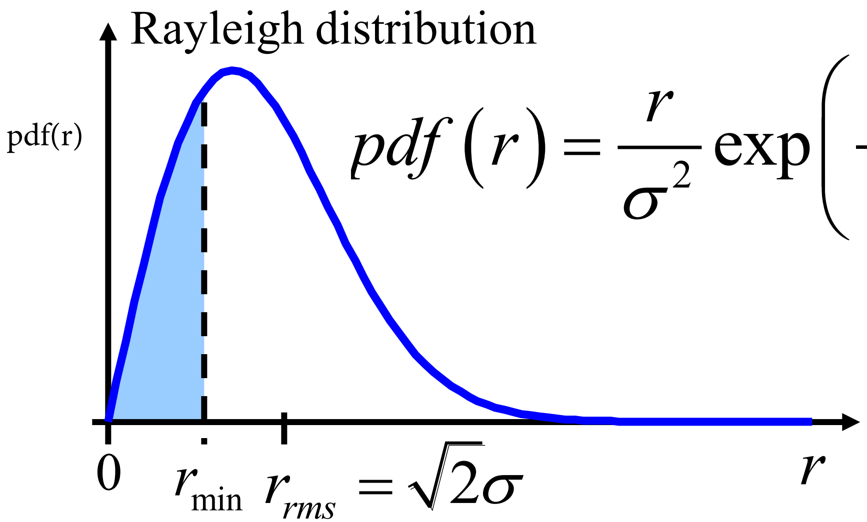
Rayleigh



No line-of-sight component

$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

Small-scale fading / Rayleigh fading



$$pdf(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

See Section 5.4.3 for the properties of the Rayleigh Distribution. It is a worst-case scenario that is a function of only one parameter, the mean received power.

The cumulative distribution function (cdf) is defined as the probability that a random variable (r) has a value smaller than r min which is the integral of the pdf. For our situation this is the Rayleigh pdf (Figure 5.15)

$$\underset{\text{failure case}}{\Pr\left(r < r_{\min}\right)} = \int_0^{r_{\min}} pdf(r)\,dr = 1 - \exp\left(-\frac{r_{\min}^{\;2}}{r_{rms}^{\;2}}\right) = \text{cdf}(r)$$

Small-scale fading
Rayleigh fading – fading margin

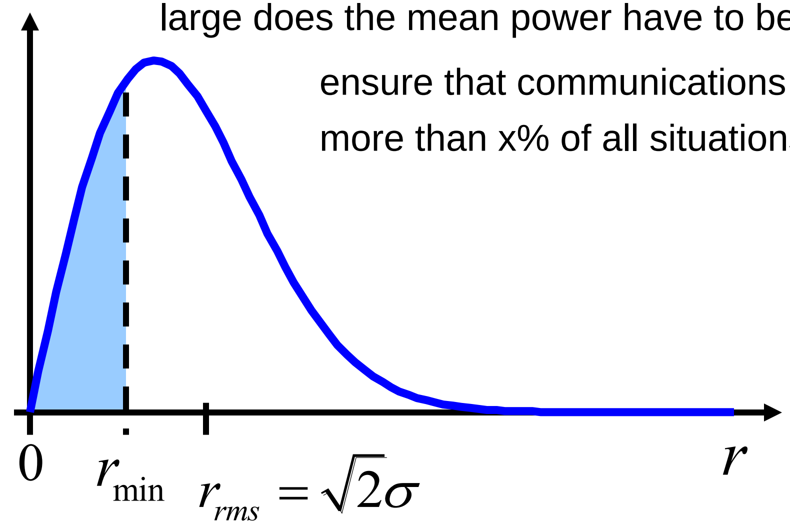
To insure success of the signal with respect to the mean power level (the RMS value)

Fading Margin M

$$M = \frac{r_{rms}^{2}}{r_{\min}^{2}}$$

$$M_{|dB} = 10\log_{10}\left(\frac{r_{rms}^{2}}{r_{\min}^{2}}\right)$$

See Section 5.4.4 Page 82 for more details on fading statistics. The fading margin answers the question, "Given a min rx power for comm, how large does the mean power have to be in order to ensure that communications fails in no more than x% of all situations



Signals handled from a Stochastic view. Rayleigh Distribution is a worst case scenario with no dominant signal component.

Small-scale fading
Rayleigh fading – fading margin

How many dB fading margin, against Rayleigh fading, do we need to obtain an outage probability of 1%?

The cdf gives us the probability of failure

$$\Pr\left(r < r_{\min}\right) = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) = 1\% = 0.01$$

Some manipulation gives

$$1 - 0.01 = \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) \Rightarrow \ln\left(0.99\right) = -\frac{r_{\min}^2}{r_{rms}^2}$$

$\ln_e = -0.0100503 = -0.01$

How large does the signal need to be with respect to the mean power level?

$$\Rightarrow \frac{r_{\min}^2}{r_{rms}^2} = -\ln\left(0.99\right) = 0.01 \quad \Rightarrow M = \frac{r_{rms}^2}{r_{\min}^2} = 1/0.01 = 100$$

actually 99.499

$$\Rightarrow \boxed{M_{|dB} = 20}$$

The signal reserve we must have in order to keep the outage probability < 1% for a Rayleigh fading scenario

Small-scale fading
Rayleigh fading – signal and interference

- What is the probability that the instantaneous SIR will be below 0 dB if the mean SIR is 5 dB when both the desired signal and the interferer experience Rayleigh fading?

$$\Pr\left(r < r_{\min}\right) = 1 - \frac{\overline{\sigma}^2}{(\overline{\sigma}^2 + r_{\min}^2)} = 1 - \frac{3.163}{3.163 + 1} \approx 0.24 \text{ or } \mathbf{24\%}$$

Equation 5.25 on page 82 --> cdf(r)

The cdf is the $\Pr(r < r_{min})$. The cdf is the integral of the pdf where this pdf is the ratio of two random variables (signal σ_1 and interference σ_2) each of which is Rayleigh distributed (LOS signal components blocked on both signals). The pdf is Equation 5.24 on page 82.

SIR = Signal σ_1 to Interference σ_2 ratio

$\overline{\sigma}^2 = (\overline{\sigma}_1)^2 / (\overline{\sigma}_2)^2$ = the ratio of the mean signal power to the mean interference power

= 5 dB = 3.163 *since 10 log (3.163) = 5 dB*

$(r_{\min})^2$ = 0 dB = 1 *since 10 log (1) = 0 dB*

This type of calculation is used to calculate the reuse distance in a cellular system; that is, how far the cells need to be apart that are using the same frequency. Chapter 3 page 44 and Chapter 17 page 379.

See Example 5.1 on page 82 for another worked problem

An approximation for $\Pr(r<r_{\min}) = r^2_{\min} / 2\,\sigma^2$ Eq 5.21

for this example the approximation = 1/(2x3.163) = 16%

Small-scale fading
one dominating component

In case of Line-of-Sight (LOS) one component dominates. (the direct path)

- Assume it is aligned with the real axis

$$\mathrm{Re}(r) \in N(A, \sigma^2) \quad \mathrm{Im}(r) \in N(0, \sigma^2)$$

- The received amplitude has now a Ricean distribution instead of a Rayleigh (the Rayleigh distribution is a subset of a Ricean distribution)

  Addition of a dominant signal line-of-sight (LOS) component reduces the probability of deep fades as experienced with Rayleigh-fading for the no LOS signal components

- The ratio between the power of the LOS component and the diffuse components is called Ricean K-factor

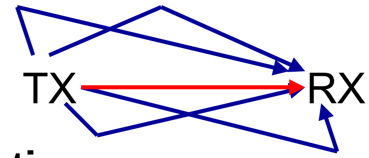
$$k = \frac{\text{Power in LOS component}}{\underset{\text{(diffuse parts)}}{\text{Power in random components}}} = \frac{A^2}{2\sigma^2}$$

for k --> 0 the Ricean distribution becomes the Rayleigh distribution

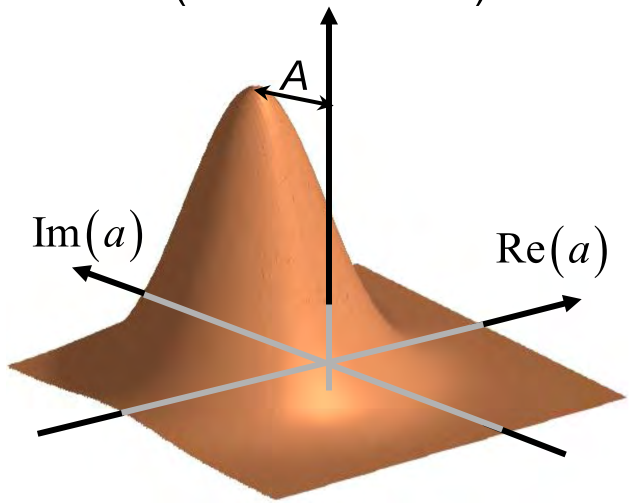
Small-scale fading / Rice fading

compare to slide 108 for Rayleigh distribution which has a zero mean and k = 0

A dominant component (line of sight)



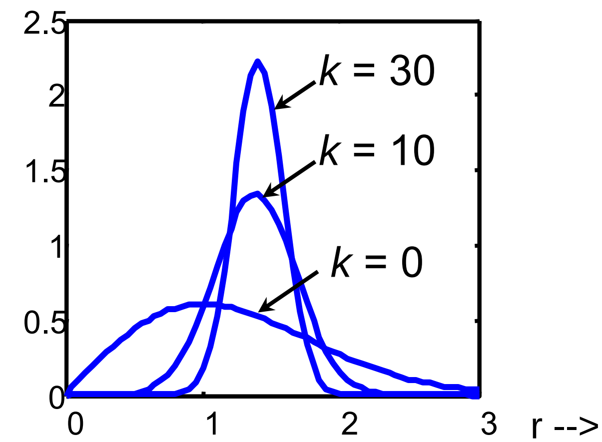
Tap distribution
2D Gaussian
(non-zero mean)



$$r = |\alpha|$$

Line-of-sight (LOS) component with amplitude *A*.

Amplitude distribution



Eq 5.27

$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{rA}{\sigma^2}\right)$$

Io is a Bessel function of the 1st kind, zero order

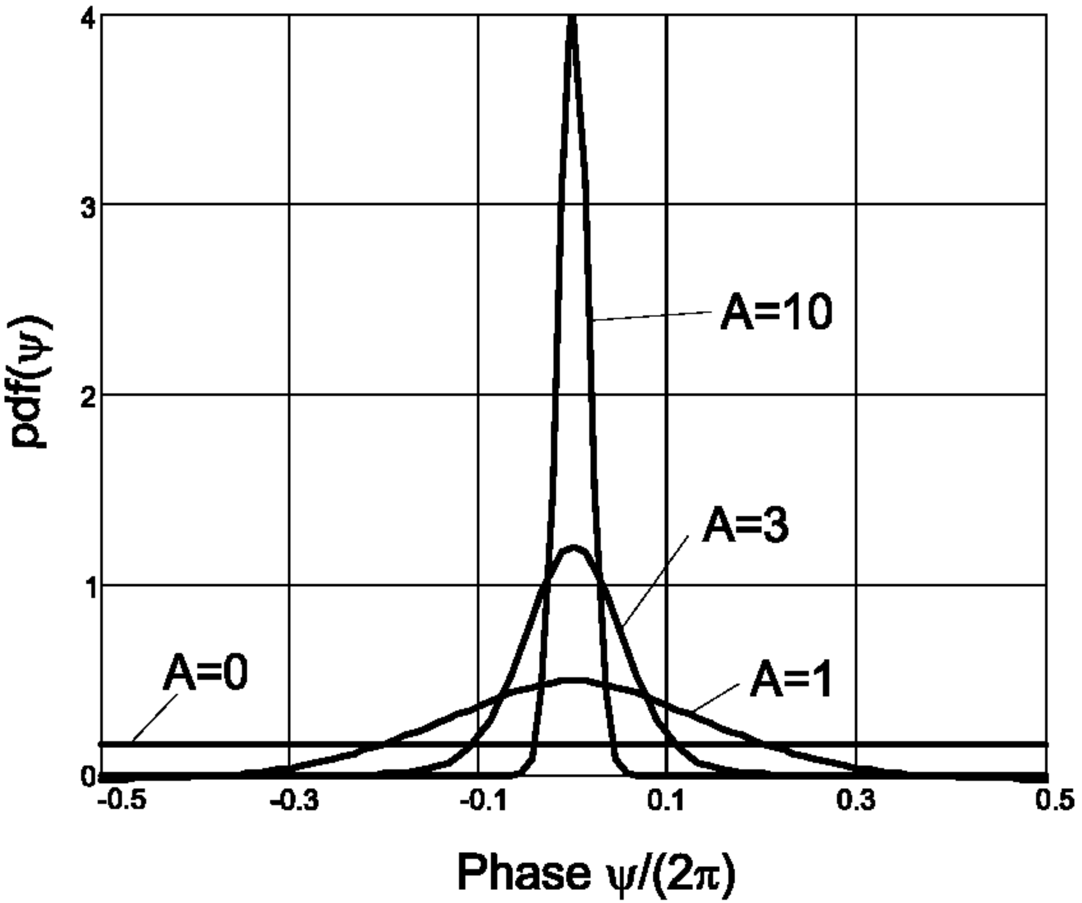
Small-scale fading
Rice fading, phase distribution

pdf of the phase of a non-zero mean Gaussian Distribution

Shown for a mean = 1 with various values for the dominant component amplitude A showing that the phase of the total signal is close to phase of the dominant signal

As compared to a zero-mean Gaussian Distribution where the pdf of the phase was a uniform distribution (Eq 5.15) thus not a factor in the Rayleigh distribution of the signal amplitude r

Mathematical work done before (1947) the advent of wireless communications.



Small-scale fading
Nakagami distribution

- In many cases the received signal can not be described as a pure LOS + diffuse components
- The Nakagami distribution is often used in such cases

$$pdf(r) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m r^{2m-1}\exp\left(-\frac{m}{\Omega}r^2\right)$$

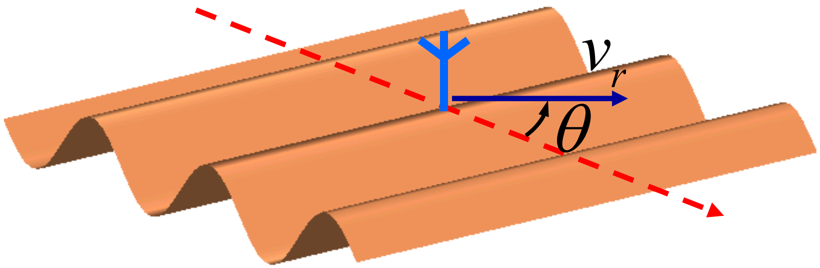
$\Gamma(m)$ is the gamma function

$$\Omega = \overline{r^2}$$

$$m = \frac{\Omega^2}{(r^2 - \Omega)^2}$$

with m it is possible to adjust the dominating power

Small-scale fading / Doppler shifts



Receiving antenna moves with speed v_r at an angle θ relative to the propagation direction of the incoming wave, which has frequency f_0

c = speed of light in a vacuum is 3 x 10^8 meters/sec

Frequency of received signal:

$$f = f_0 + \nu$$

where the Doppler shift is

$$\nu = -f_0 \frac{v_r}{c} \cos(\theta)$$

cos (0^0) = + 1   cos (180^0) = -1

The maximal Doppler shift is

$$\nu_{\max} = f_0 \frac{v}{c}$$

Small-scale fading
Doppler shifts

How large is the maximum Doppler frequency at pedestrian speeds for 5.2 GHz WLAN and at highway speeds for GSM 900?

$$\nu_{max} = f_0 \frac{v}{c}$$

Doppler Shift + or - from the carrier frequency depending on the direction of travel; moving toward each other +

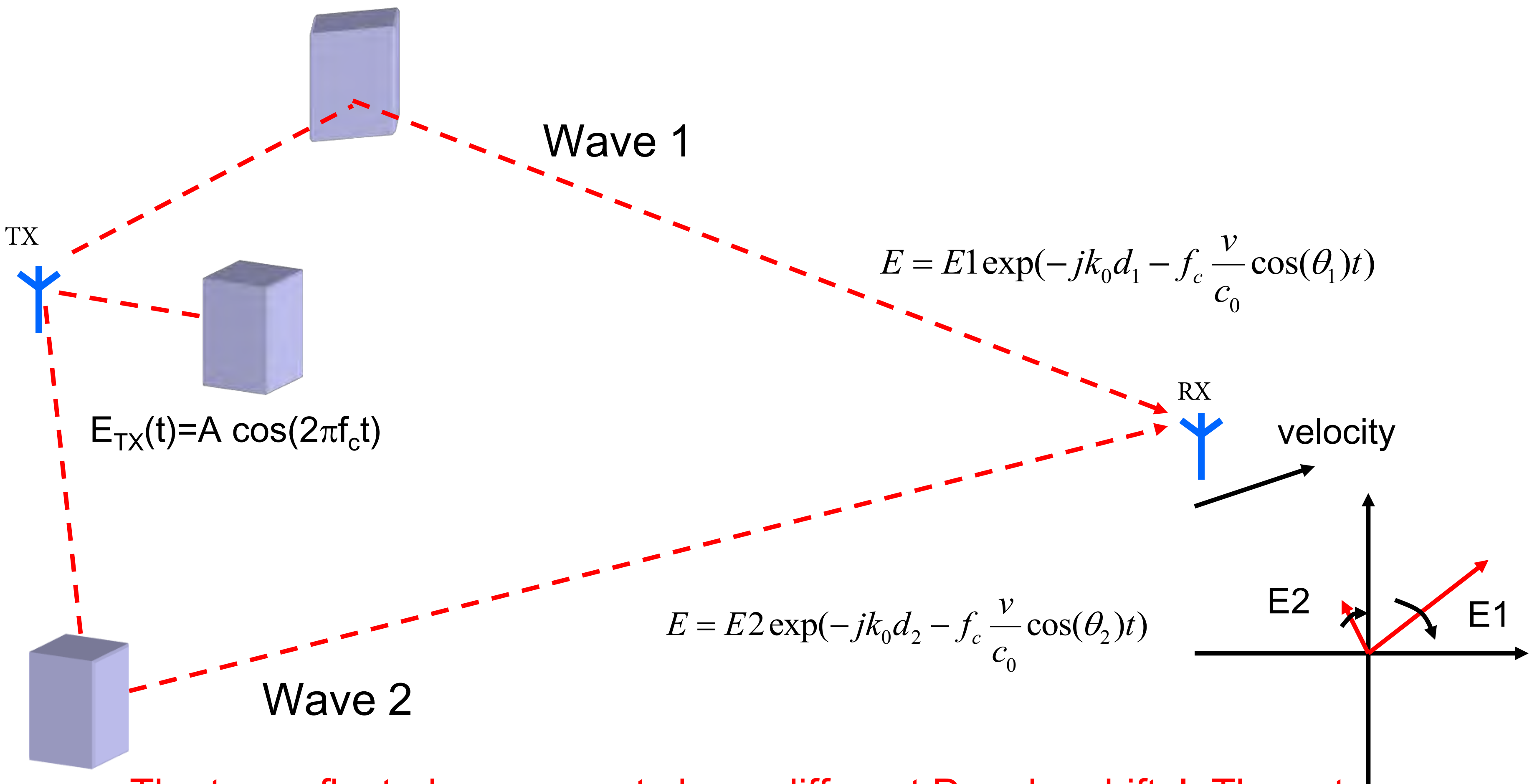
converting km/h to m/s   results in

- $f_0 = 5.2\text{x}10^9$ Hz, v=5 km/h = 1.4 m/s ⇨ 24 Hz maximum Doppler Frequency

$v_{max} = (5.2\text{x}10^9 \text{ Hz})(1.4 \text{ m/s}) / (3\text{x}10^8 \text{ m/s}) = 24.267 \text{ Hz}$

- $f_0 = 900\text{x}10^6$ Hz, v=110 km/h = 30.6 m/s ⇨ 92 Hz

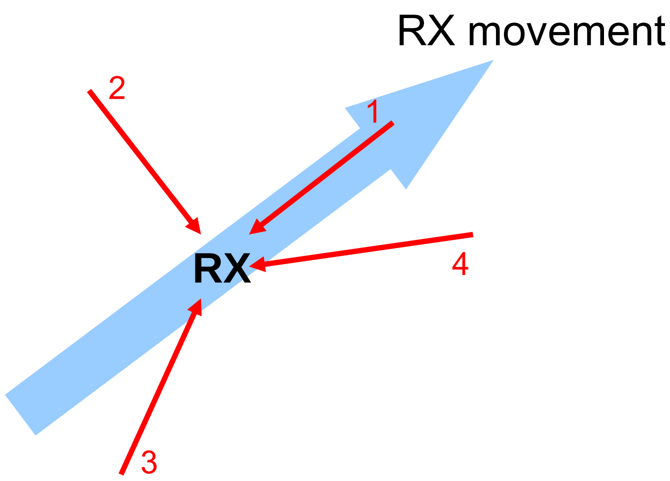
Small-scale fading
Doppler spectra



The two reflected components have different Doppler shifts! These two different Doppler shifts will cause a random frequency modulation at Rx

Small-scale fading
Doppler spectrum

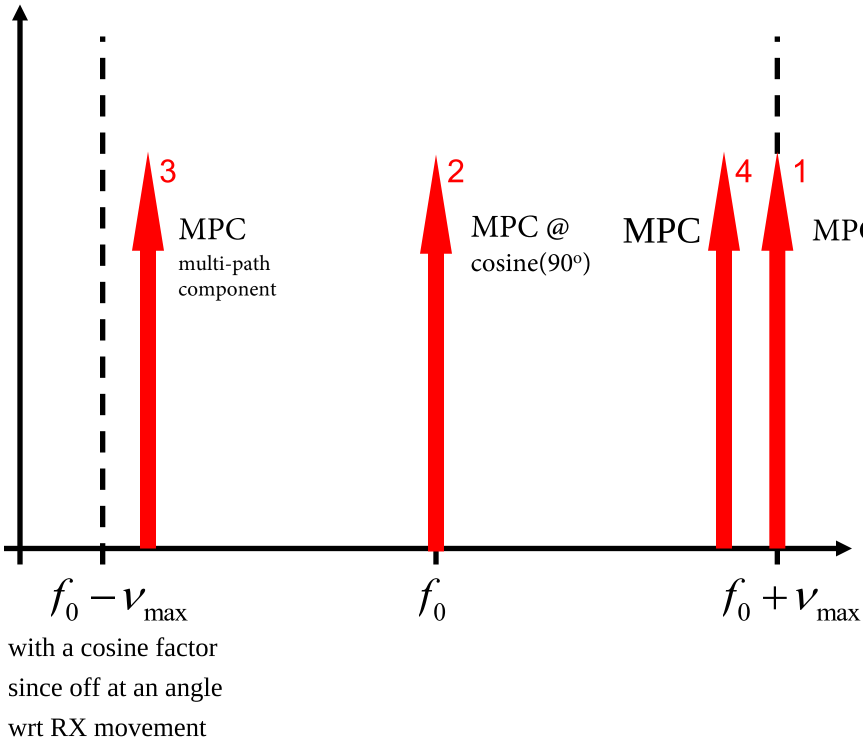
Incoming waves from several directions
(relative to movement of RX)



All waves of equal strength in this example, for simplicity.

Related to Clarke's Model, a popular statistical model for flat fading.

Spectrum of received signal when a f_0 Hz signal is transmitted.



Small-scale fading
The Doppler spectrum

Describes frequency dispersion which impacts narrowband systems like OFDM but no direct impact on most wideband systems like CDMA. But it is also a measure for the temporal variability of the channel and is important for ALL systems both wide and narrow band.

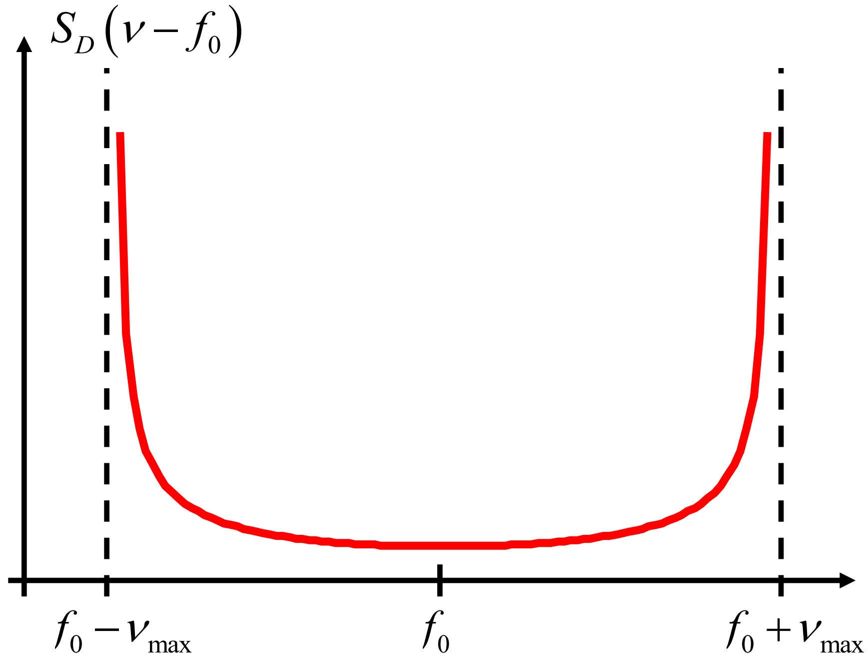
Doppler Spectrum (spectral broading)

$$S_D(\nu) = \int \rho(\Delta\tau) e^{-j2\pi\nu\Delta\tau} d\Delta\tau$$

$$\propto \frac{1}{\pi\sqrt{\nu_{\max}{}^2 - \nu^2}}$$

for $-\nu_{\max} < \nu < \nu_{\max}$

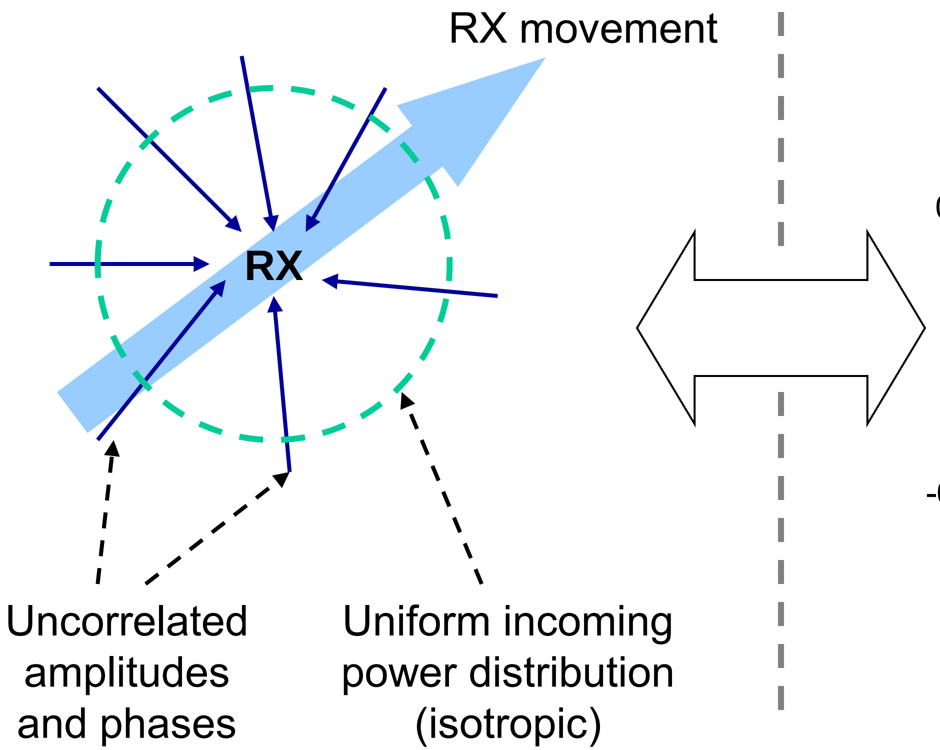
Doppler spectrum (Jakes Spectrum) at center frequency f_0.



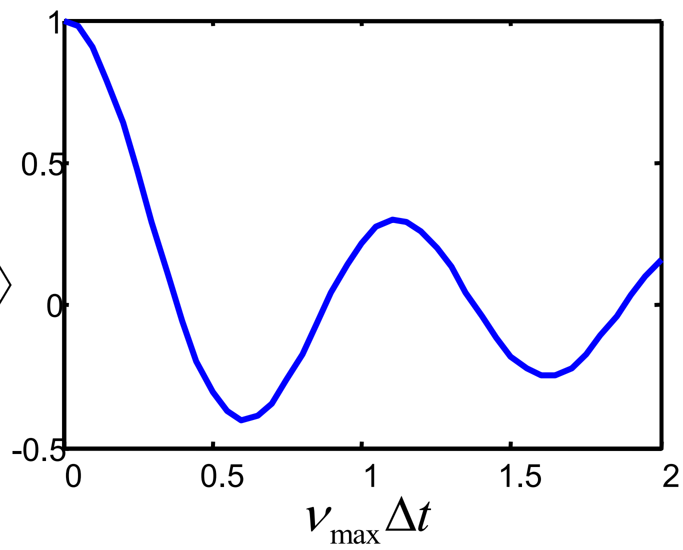
Large number of interference objects (IOs) from a uniform azimuthal distribution with no LOS component that results in a highly non-uniform Doppler spectrum. Vertical Antenna See errata sheet for full description of assumptions leading to the Doppler Spectrum equations

Small-scale fading
Doppler spectrum

Isotropic uncorrelated scattering



Time correlation



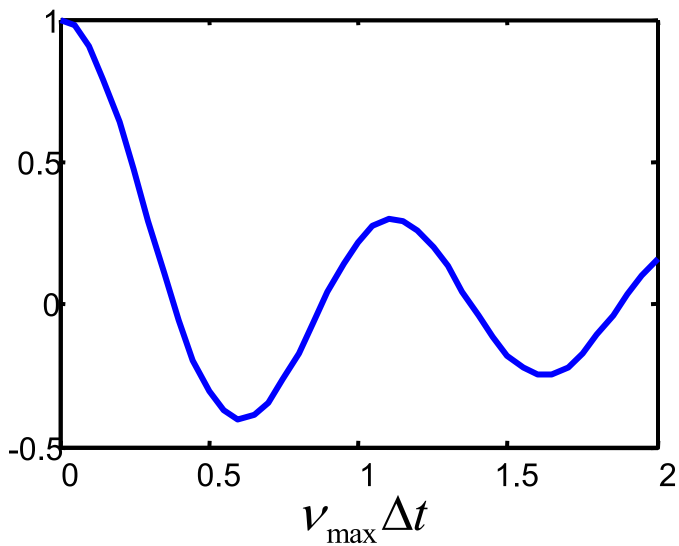
Small-scale fading / Doppler spectrum

- Time correlation – how static is the channel?

Stationary Mobile Station (MS) can still have a dynamic channel due to moving interferring objects (IOs)

$$\rho(\Delta t) = E\{a(t)a^*(t+\Delta t)\} \propto J_0(2\pi v_{\max}\Delta t)$$

Bessel Function
zero^th-order



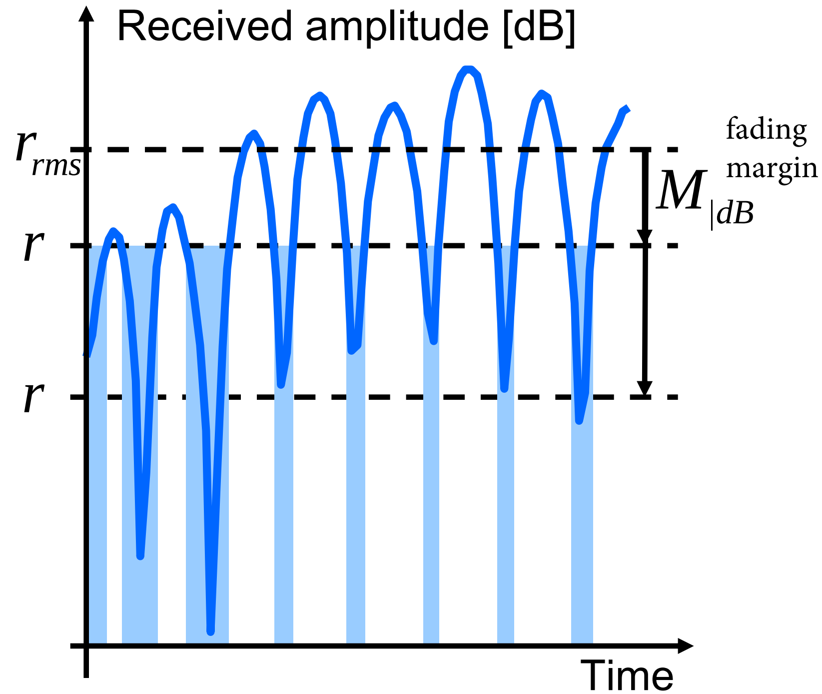
- The time correlation for the amplitude is

$$\rho(\Delta t) \propto J_0^2(2\pi v_{\max}\Delta t)$$

See example 5.3 on page 90 for a related problem dealing with a stationary MS that moves a little to deal with fading dips.

Small-scale fading / Fading dips

What about the length and the frequency of fading dips ?



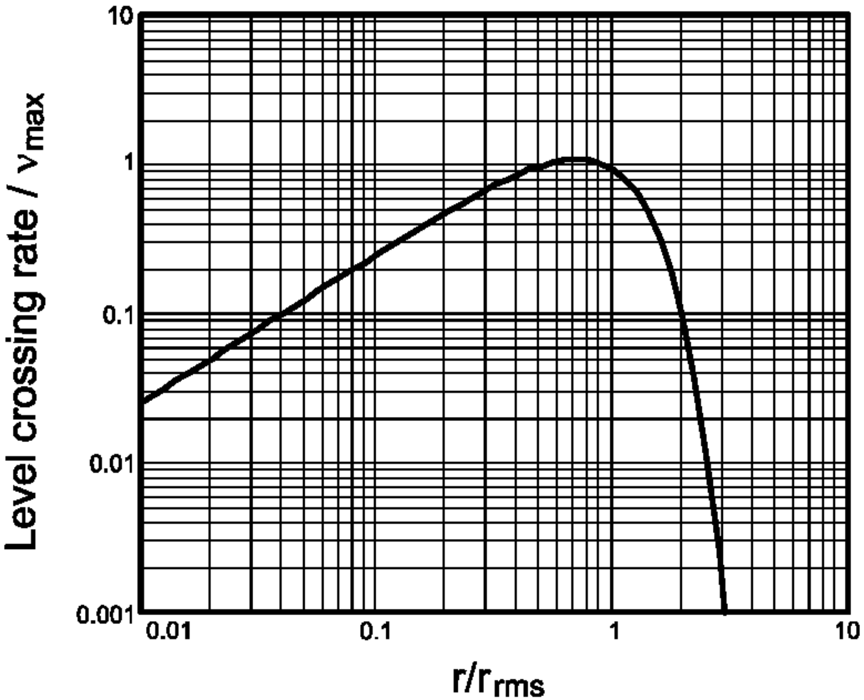
Average duration of fades (ADF)
Occurrence rate is described in the Level Crossing Rate (LCR) - a rate of falling below some selected signal level (r*) like 30 dB below the mean signal level (r_{rms})

From Section 5.7.3 on Page 91, fading dips mean lower signal level which leads to higher susceptibility to noise in addition to being related to intersymbol interference (ISI - Chapter 2). Also, the fading dips increase the probability of random frequency modulation (FM) which results in errors for any system that conveys information by means of the phase of the transmitted signal (PSK, MPSK, etc.)

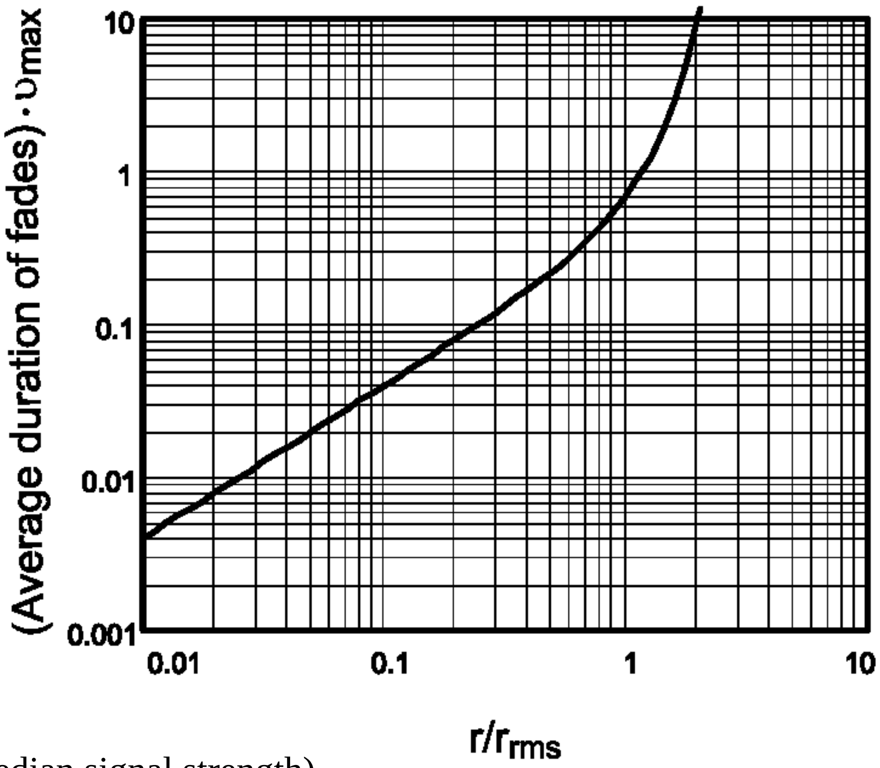
Small-scale fading / Statistics of fading dips

Derivation in Appendix 5.C for Rayliegh Fading amplitude & Jakes spectrum

Frequency of the fading dips
(normalized dips/second)

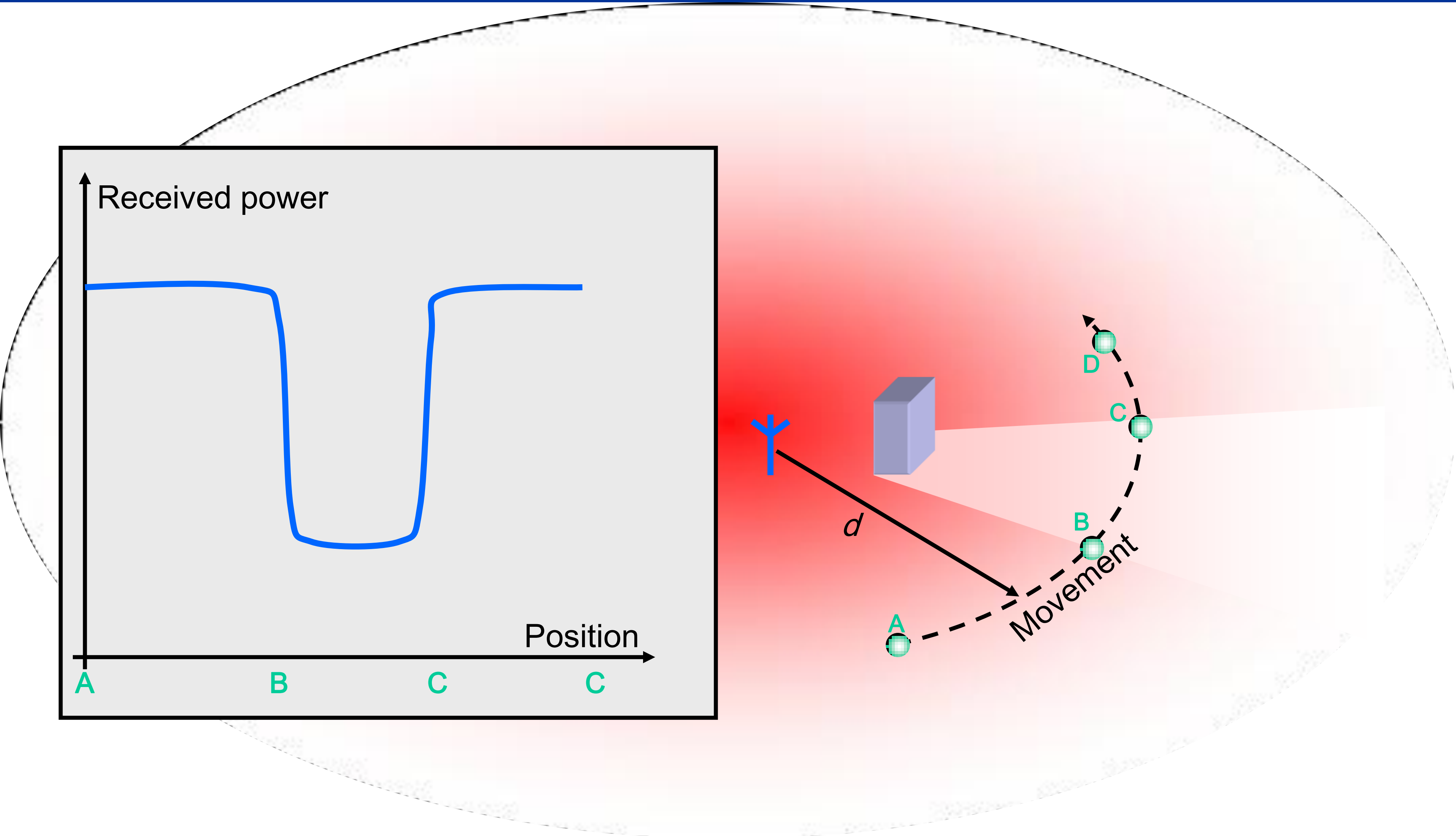


Length of fading dips
(normalized dip-length)



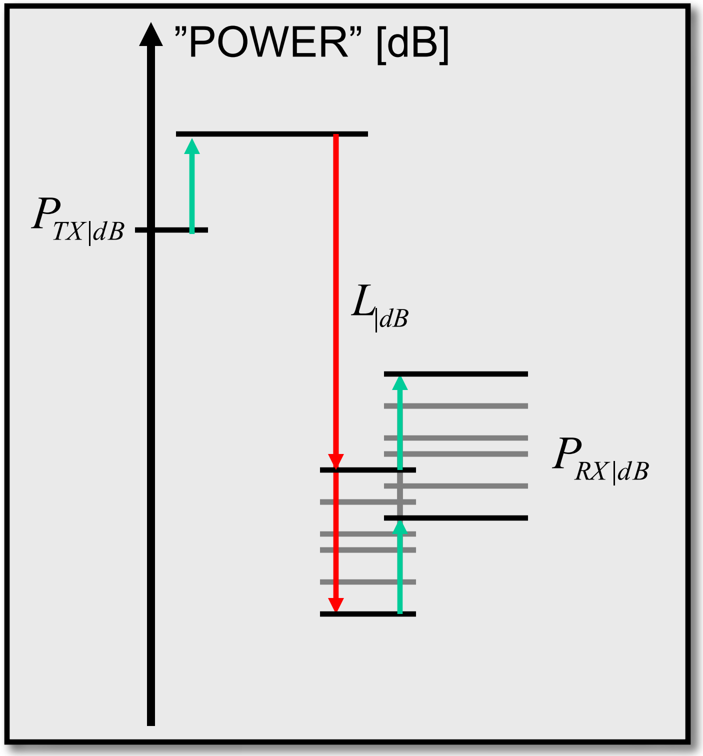
x-axis is increasing signal strength r (normalized to median signal strength)

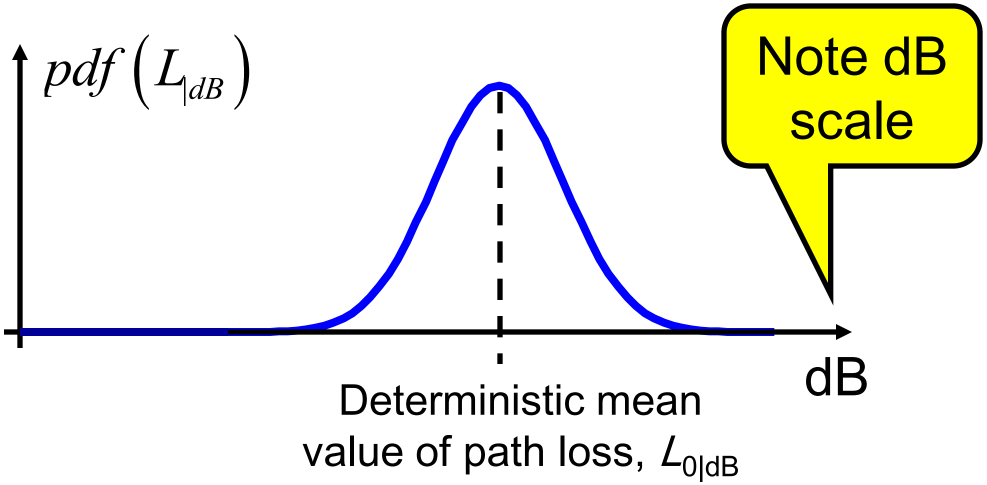
Refresher: Basic Principle of Large-scale Fading



Large-scale fading
Log-normal distribution

The fading margin becomes the sum of the margin for Rayleigh fading and large-scale fading (shadowing) that are not necessarily related even when being blocked by a large object. Thus M must take both into account (addition or Suzuki distribution/approximations Eqs 5.71/5.72 page 97)





$$pdf\left(L_{|dB}\right) = \frac{1}{\sqrt{2\pi}\sigma_{F|dB}} \exp\left(-\frac{\left(L_{|dB} - L_{0|dB}\right)^2}{2\sigma_{F|dB}^{\;2}} \right)$$

Standard deviation $\sigma_{F|dB} \approx 4 \ldots 10 \text{ dB}$