Orthogonal Frequency Domain Multiplexing

OFDM (not new technology, circa 1950)

Contents

- Principle and motivation with desire for high data rates (short symbol times coupled with large channel bandwidths necessary to handle high data rates)
- Analogue and digital implementation
- Frequency-selective channels: cyclic prefix to reduce residual delay dispersion
- Channel estimation
- Peak-to-average ratio
- Inter-channel interference
- Adaptive modulation
- Multi-carrier CDMA

PRINCIPLE, MOTIVATION AND BASIC IMPLEMENTATION

Principle (1)

- For very high data rates, equalization and Rake reception becomes difficult
  - Product of maximum excess delay (with respect to small symbol duration) and large system bandwidth makes high data rates tough
  - Especially critical for wireless LANs and PANs

Although delay dispersion of the channel is outside of our control, as the symbol duration becomes very small for high data rates then the impulse response of the channel becomes very long in terms of symbol durations. So even though we can't control channel delay dispersion, we can control the transmission system.

- Solution:
  - transmit multiple data streams with lower rates on several carriers
  - Have carriers multiplexed in the most efficient possible way:
  - Signals on the carriers can overlap and stay orthogonal

multiple data streams at lower data rates results in increased the symbol duration times but one still needs to separate the different signals from each other which requires that all of the signals be orthogonal.

Principle (2)

- How close can we space the carriers?

$$f_n = nW/N \qquad W = N/T_S$$

N distinct carriers for N parallel data streams
fn = subcarrier frequencies   n = integer
Ts = symbol duration
W = total bandwidth available

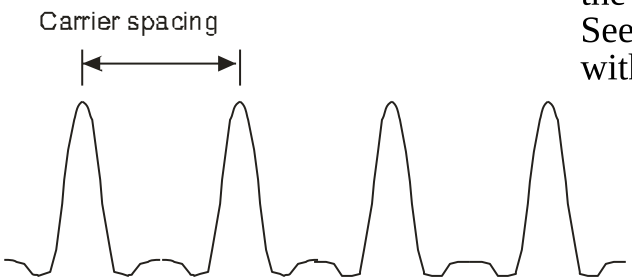
- Carriers are still orthogonal

$$c_n c_k \int_{iT_S}^{(i+1)T_S} \exp(j2\pi f_n t)\exp(-j2\pi f_k t)dt = c_n c_k \delta_{nk}$$ zero correlation if n not equal to k

Each of the fn subcarriers is pulse modulated by the data source fk - a binary/rectangular pulse. See Figure 11.2 for the spectrum of a rectangular pulse with its sin(x) / x characteristic shape

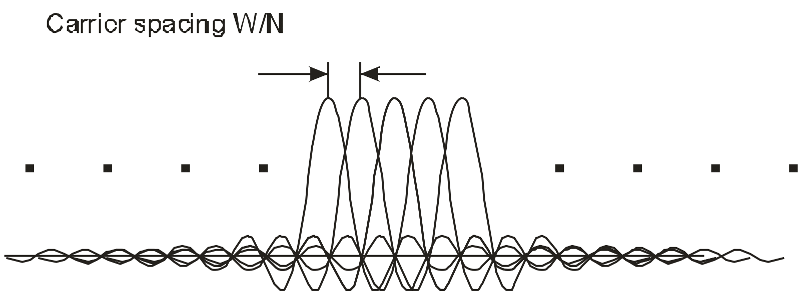
Frequency Domain

FDMA



One could use FDMA but it has a large bandwidth which wastes spectrum

OFDM



Spectrum of all the fn subcarriers where the data source is a binary 1 for each of the N data streams.
Each carrier is in the spectral nulls of all the other carriers - no interference

OFDM Orthogonality

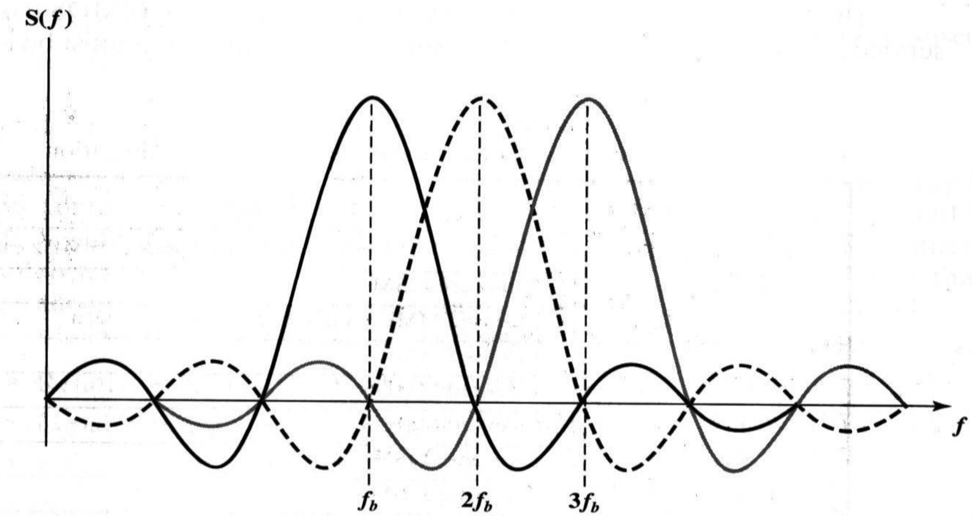
- OFDM Signal in the Frequency Domain showing where the peak of one subcarrier is in the null of another (eliminating interference)
- Subcarrier Spacing

  $f_b = iW/N$ where i = integer

  $W = N/T_s$ W = total bandwidth available

  T_s = symbol duration

  N = # of distinct carriers for N parallel data streams

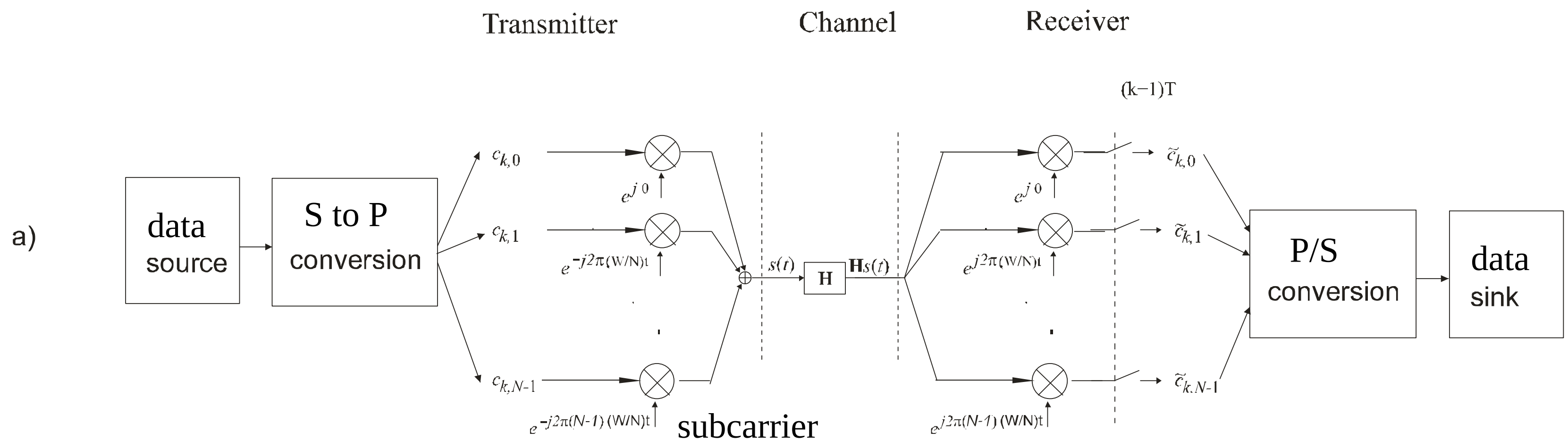


(b) Three subcarriers in frequency domain

Figure 11.12 Illustration of Orthogonality of OFDM

Analogue vs. digital implementation (Fig 19.2 Page 419)

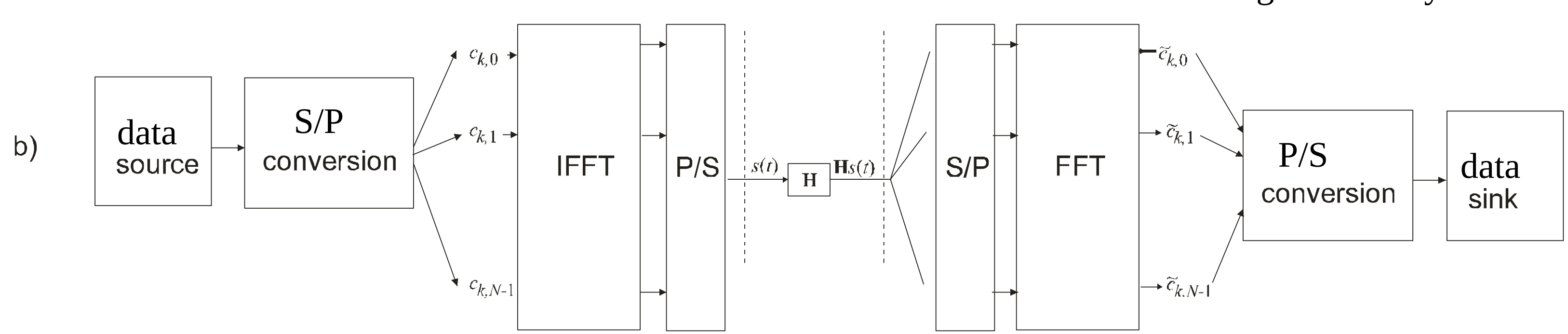
- Analog implementation (very difficult/expensive to implement the subcarrier oscillators in hardware)



subcarrier
oscillators fn

- Digital implementation

Receiver demodulation - for each k, multiply by exp(-j 2pi f t) and integrate over symbol duration Ts



the n values of the IFFT (for each fn) have to be transmitted one after the other which requires the P/S conversion after the IFFT

Why can we use an IFFT?

- Transmit signal is Looking at the analog implementation

$$s(t) = \sum_{i=-\infty}^{\infty} s_i(t) = \sum_{i=-\infty}^{\infty} \sum_{n=0}^{N-1} c_{n,i} g_n(t - iT_S)$$

Each index i corresponds to a pulse (temporal data), each index n to a carrier frequency

- With basis pulse (normalized, frequency shifted rectangular pulse PAM)

$$g_n(t) = \begin{cases} \frac{1}{\sqrt{T_S}} \exp(j2\pi n \frac{t}{T_S}) & \text{for } 0 < t < T_S \\ 0 & \text{otherwise} \end{cases}$$

- Transmit signal sampled at $t_k = kT_S/N$

$$s_k = s(t_k) = \frac{1}{\sqrt{T_S}} \sum_{n=0}^{N-1} c_{n,0} \exp(j2\pi n \frac{k}{N}) \; .$$

- This is the definition of an IFFT

actually the Inverse Discrete Fourier Transform (IDFT) of the transmit symbols. The transmitter can be realized by the performing an IFFT on the block of transmit symbols where the blocksize is equal to the number of subcarriers and n is a power of 2

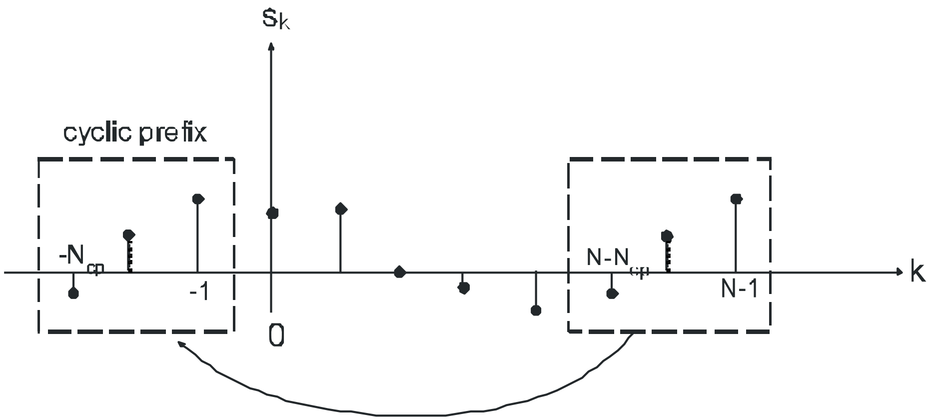
Frequency-selective channels

- Cyclic prefix, i.e., repeat last samples at beginning of symbol

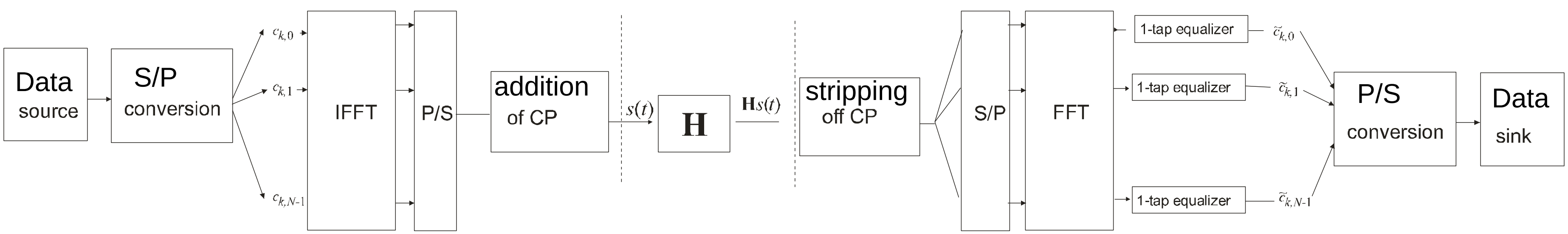
addition of CP which is a special type of guard interval - reduces residual delay dispersion

- Converts linear to circular convolution

Frequency selective channel produces delay dispersion even if the symbol duration on each OFDM carrier is longer than the delay spread This leads to loss of orthogonality between subcarriers while producing ICI

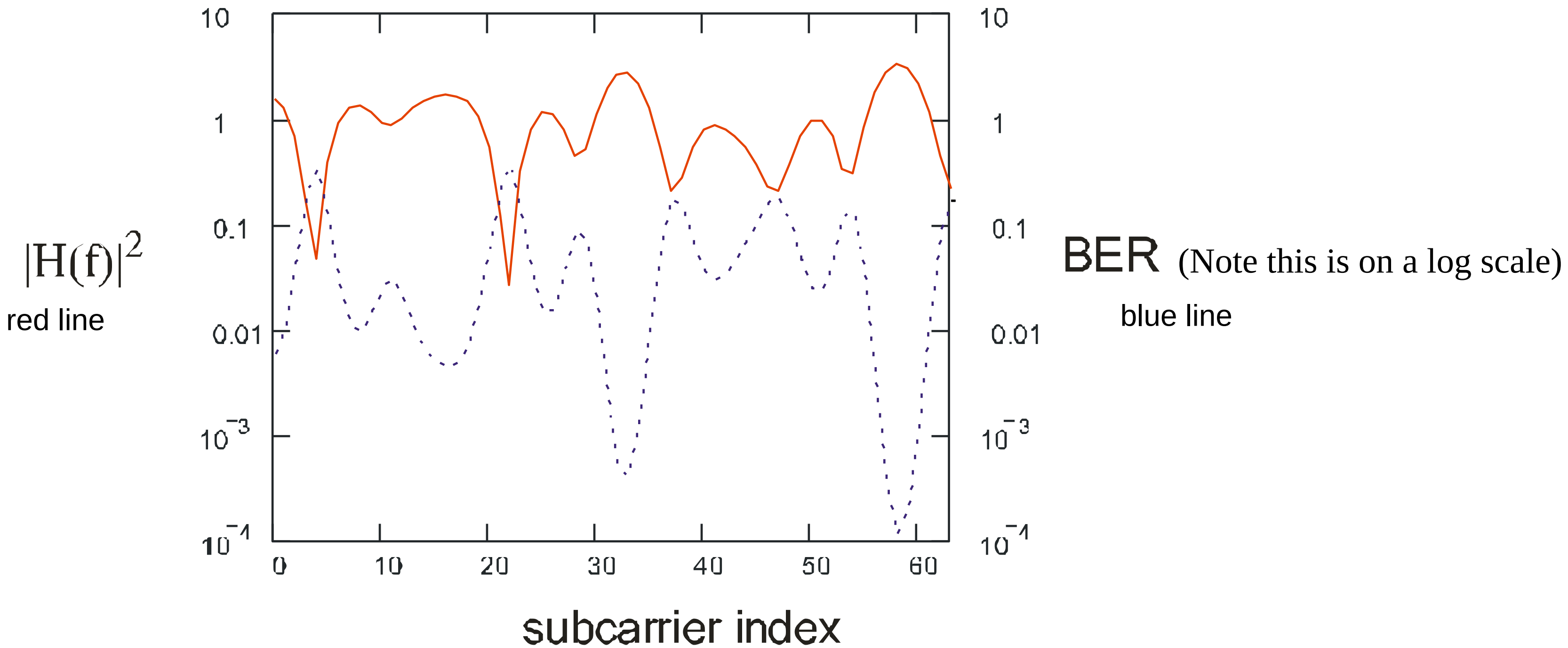


Cyclical/Circular Convolution of the transmit signal with the channel impulse response and the receive filter. The RX filter removes the first part of the signal-the cyclic prefix (10% of the symbol duration so only a small impact to the SNR and spectral efficiency at the RX). This produces a number of parallel non- dispersive (flat) channels which can be easily equalized recovering the orthogonality. See Eq 19.5 for new basis pulse that implements CP.



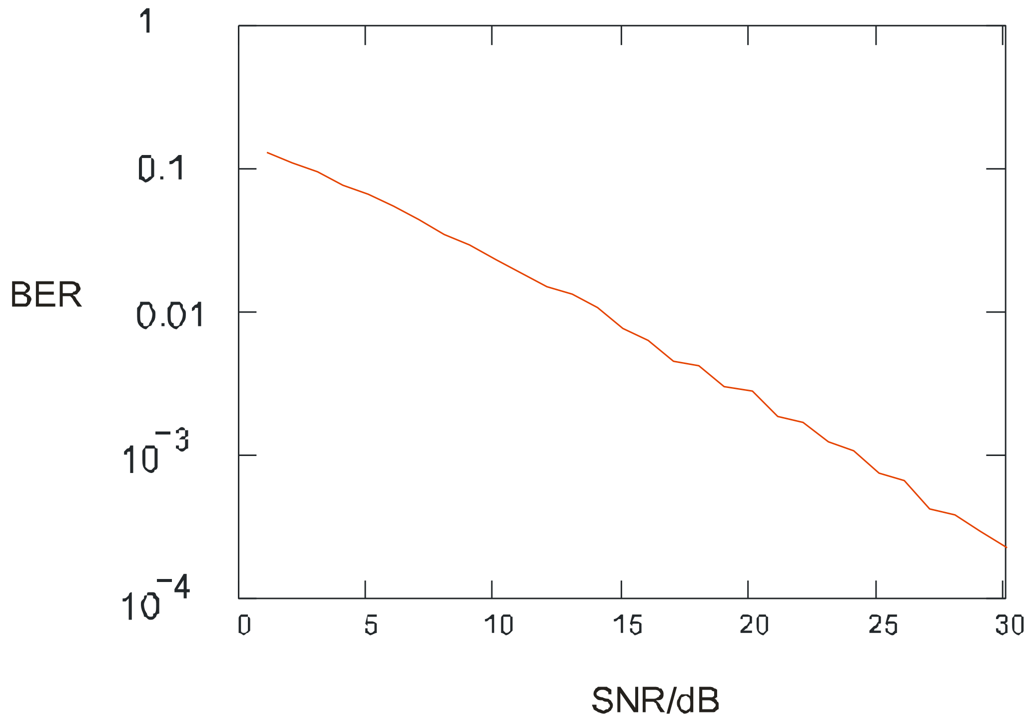
Performance in frequency-selective channels

CP converts a frequency-selective channel (w/noise) into a number of parallel flat-fading channels - no ISI
But an uncoded OFDM system with CP doesn't show any frequency diversity so if a subcarrier (fn) is in a fading dip, then the subcarrier's BER is very high and this dominates the entire system's BER



Performance in frequency-selective channels (continued)

Average BER over all of the subcarriers for a frequency selective channel showing the BER only decreases linearly as the SNR increases. Thus carriers with poor SNR dominate the OFDM system's performance.



Performance in frequency-selective channels (continued)

- How to improve performance?
  - adaptive modulation (different signal alphabets in different subcarriers)

    Stronger encoding, smaller signal alphabets for carriers with low SNR. Power allocated to each subcarrier can be increased also.

  - Coding across different tones

    coding to compensate for fading dips on one subcarrier by using another subcarrier with a good SNR

  - Spreading the signal over all tones (multicarrier CDMA)

    Each symbol is spread across all carriers so that the resultant OFDM system has an SNR that is the average of all the tones/subcarriers over which it is spread

Coding can improve performance in OFDM systems but with data being transmitted at different times on different frequencies - how? As we've seen before, interleaving works and in this case interleaving that depends on the estimated characteristics of the channel (subcarriers). See Figure 19.7 for a performance graph

ADVANCED IMPLEMENTATION ISSUES

Channel estimation

- Easiest approach: dedicated pilot symbols

Produces good results but computationaly complex of the order # of subcarriers squared N**2

- Estimated channel gain on subchannel n

$$h_{n,i}^{\mathrm{LS}} = r_{n,i}/c_{n,i}$$

where r is the received signal and c the transmit signal

- Performance improvement:
  - Channels on subcarriers are correlated
  - Exploit that knowledge for noise averaging

even if frequency selective (delay dispersion which produces fading channels) use this downside to some advantage

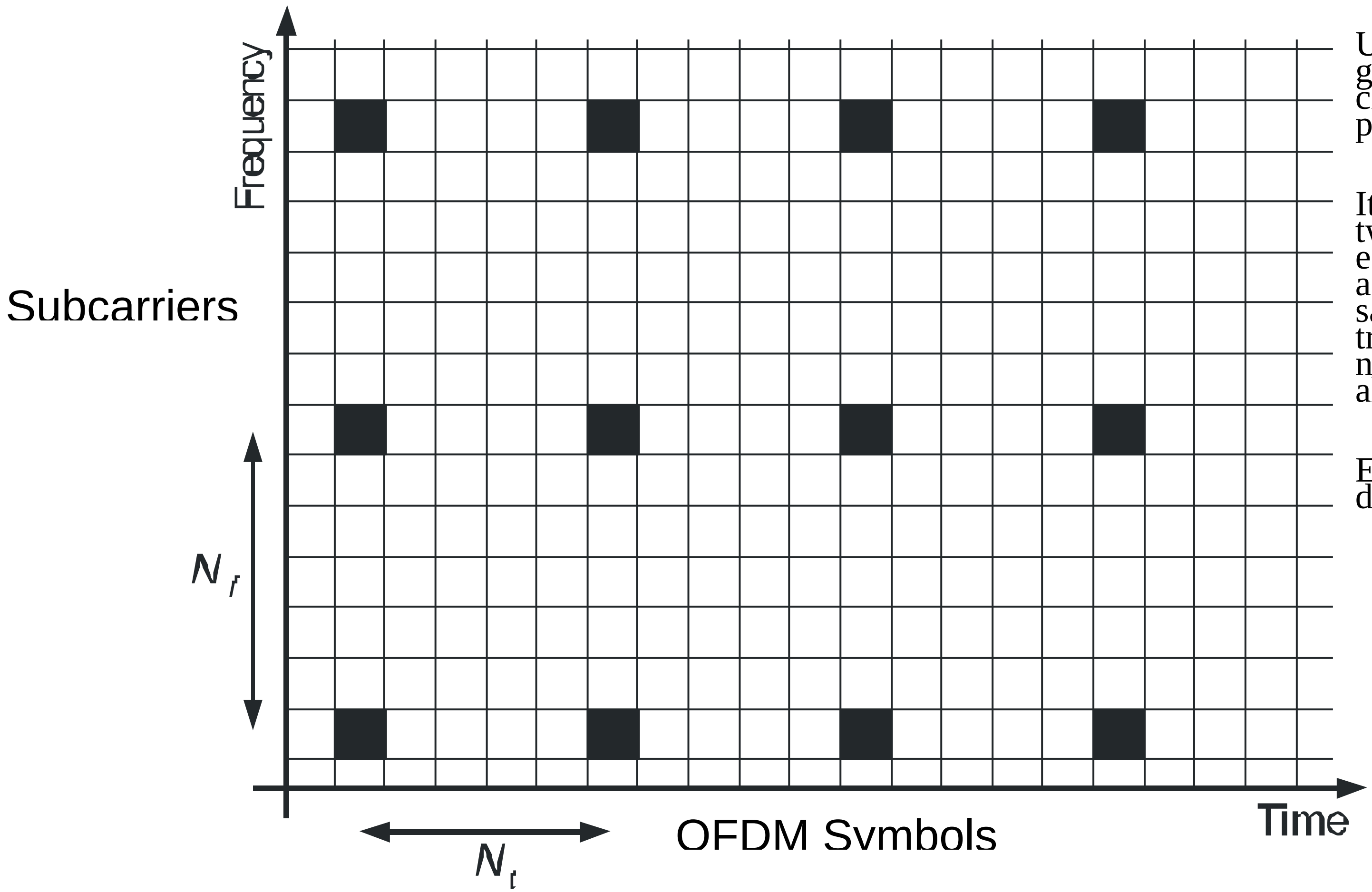
$$\mathbf{h}_i^{\mathrm{LMMSE}} = R_{hh^{\mathrm{LS}}} R_{h^{\mathrm{LS}}h^{\mathrm{LS}}}^{-1} \mathbf{h}_i^{\mathrm{LS}}$$

$R_{hh^{\mathrm{LS}}}$: covariance matrix between channel gains and least-squares estimate of channel gains,

$R_{h^{\mathrm{LS}}h^{\mathrm{LS}}}$: autocovarance matrix of least-squares estimates

Channel estimation (continued)

- Reduction of overhead by scatterered pilots



Use Estimation Theory to gain knowledge of channel characteristics between the pilot symbol placements

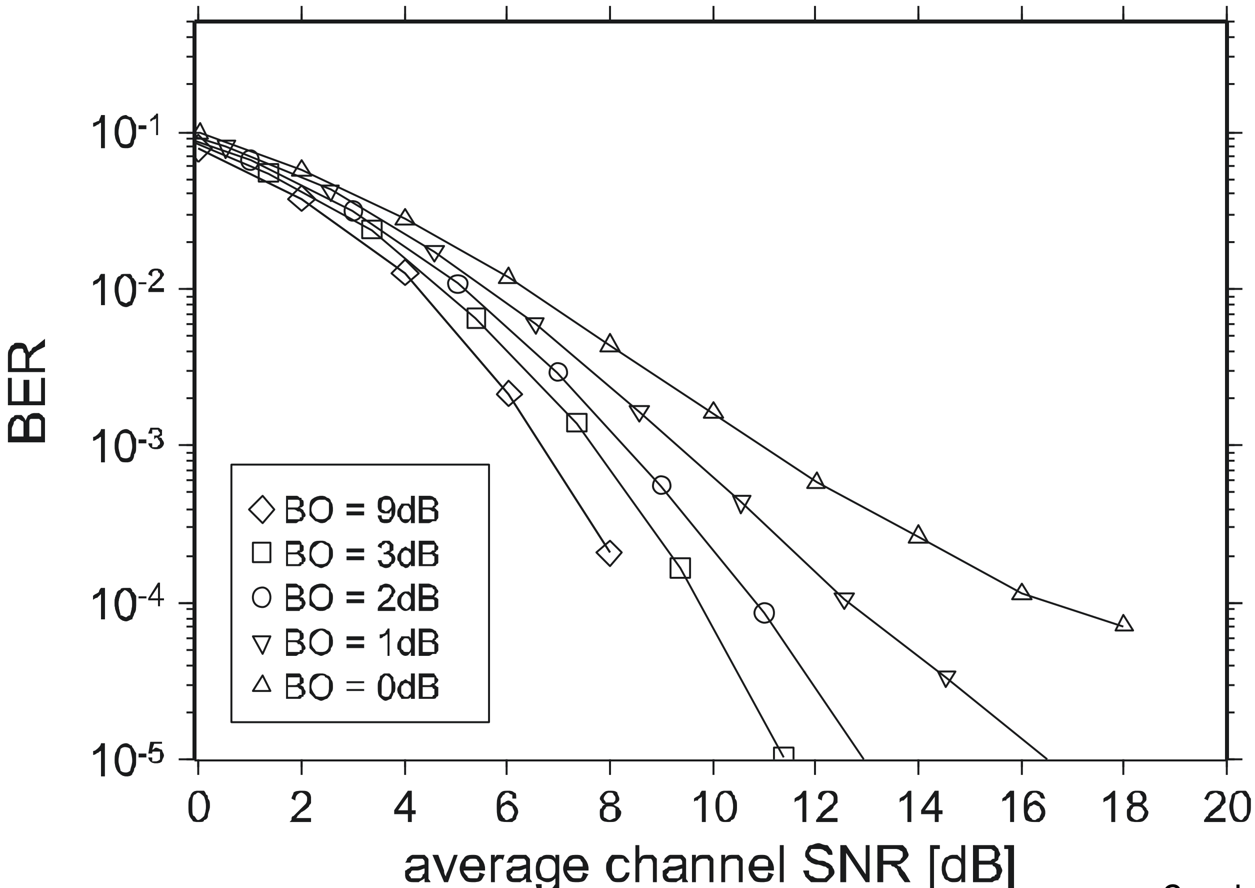
It has been found that using twice as many pilots in each direction (time, freq) as required by the sampling theorem is a good tradeoff on pilot symbol noise versus estimation algorithm complexity

Eigenvalue Decompostion discussed in Section 19.5.3

Effect of PAR Problem (Peak Average Problem)

- Increases BER

Peak Average Problem - problem for OFDM signal with its large dynamic range for all the subcarrier signal components. Use linear (Class A) transmitters but these are power consuming/not efficient and expensive



BO - backoff level of Xmtr

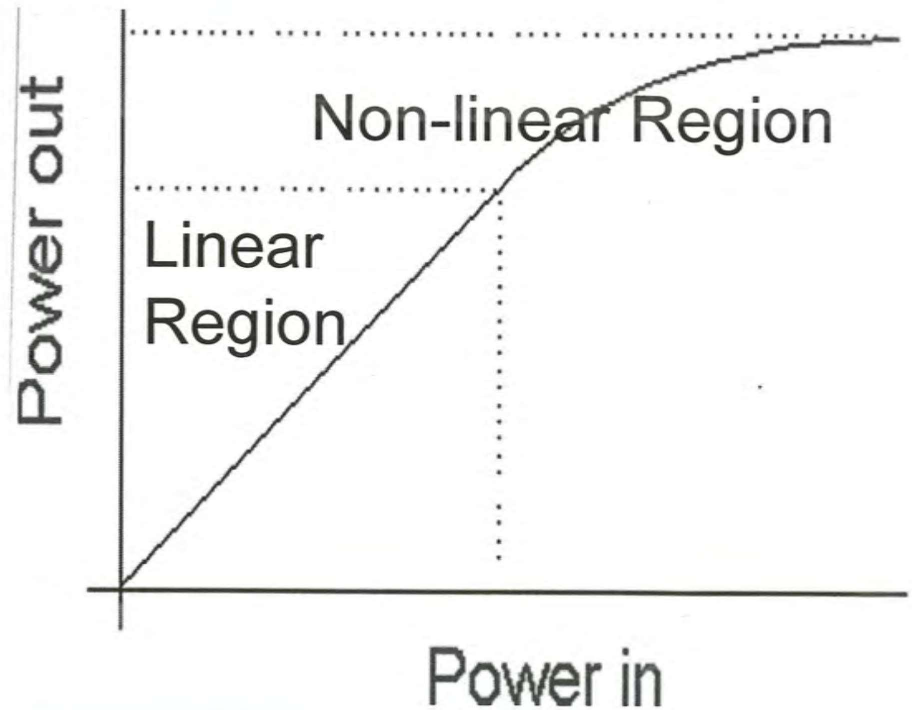
Remedies for the PAR problem (1)

- Backoff

Operate an amplifier at a fraction of it's rated maximum power since the non-linear region of operation is at the higher power levels

Compression level depends on the amplifier (Class A, E, etc.)

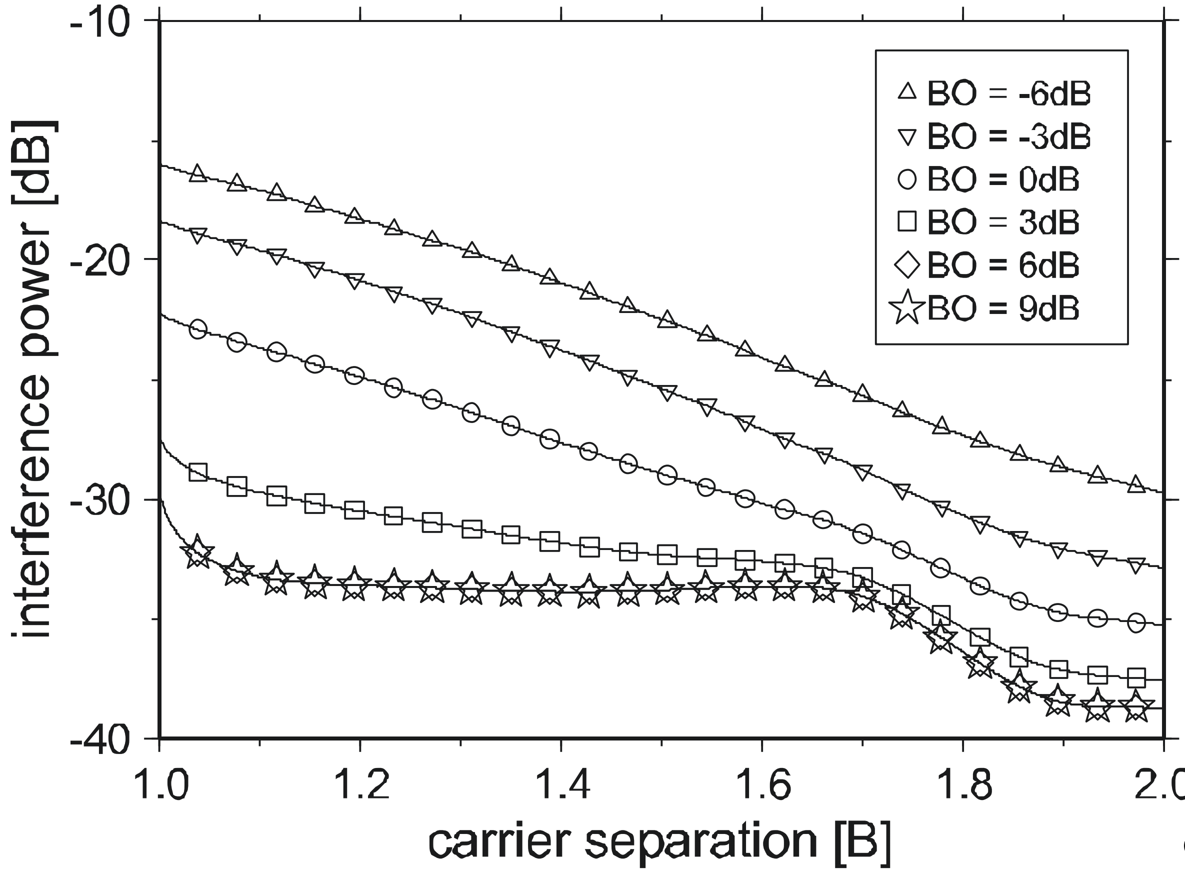
Backoff Level can range from 1 to tens of dB



Nonlinear amplifiers lead to distortion which destroys subcarrier orthogonality and produces out-of-band emissions - spectral regrowth similar to 3rd order inter-modulation products Distortion impacts BER and large bandwidth signals cause adjacent channel interference

Remedies for the PAR problem (2)

– Residual cutoff results in spectral regrowth Interference



Remedies for the PAR problem (3)

- Coding for PAR reduction

  Normally each OFDM symbol represents one codeword with 2**N possibilities assuming BPSK modulation. This PAR reduction coding is a subset of size 2**K where both the transmitter and receiver know the mapping between the code of length K and the codeword of length N which reduces PAR

- Phase adjustments
  - Cannot guarantee certain PAR

Remedies for PAR problem (4)

- Correction by multiplicative factor
  - Simplest case: clipping
  - More gentle: Gaussian functions

$$\hat{s}(t) = s(t)\left[1 - \sum_n \max\left(0, \frac{|s_k| - A_0}{|s_k|}\right) \exp\left(-\frac{t^2}{2\sigma_t^2}\right) \right]$$

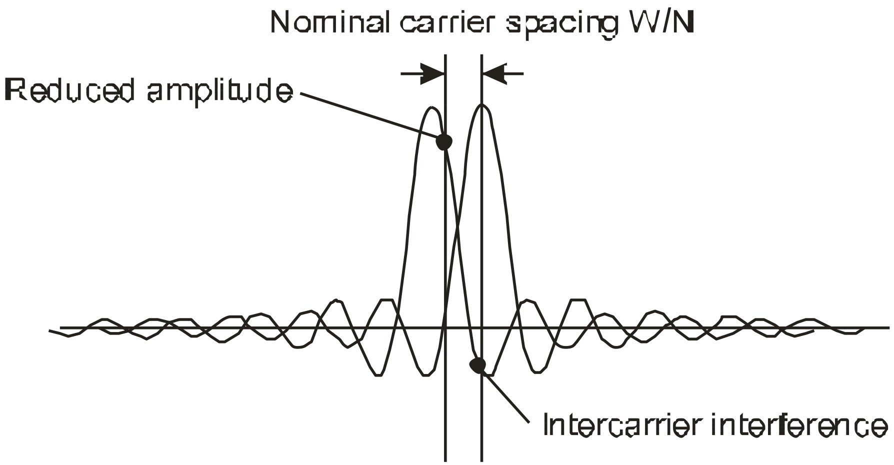
- Correction by additive factor

(see Section 19.6.2 for both of these PAR reduction techniques)

Intercarrier Interference (ICI)

- Intercarrier interference occurs when subcarriers are not orthogonal anymore

Delay dispersion leads to a loss of orthogonality between subcarriers



null of one subcarrier is no longer at the peak of the adjacent subcarrier

Remedies for ICI (1)

- Optimize the carrier spacing and symbol duration
  - Larger subcarrier spacing leads to smaller ICI
  - Larger spacing leads to shorter symbol duration: more sensitive to ICI; cyclic prefix makes it less spectral efficient
  - Maximize

$$SINR = \frac{\frac{E_S}{N_0} P_{sig} \frac{N}{N_{cp}+N}}{\frac{E_S}{N_0} P_{sig} \frac{N}{N_{cp}+N} \frac{P_{ISI}+P_{ICI}}{P_{sig}} + 1}$$

- Optimum choice of OFDM basis signals

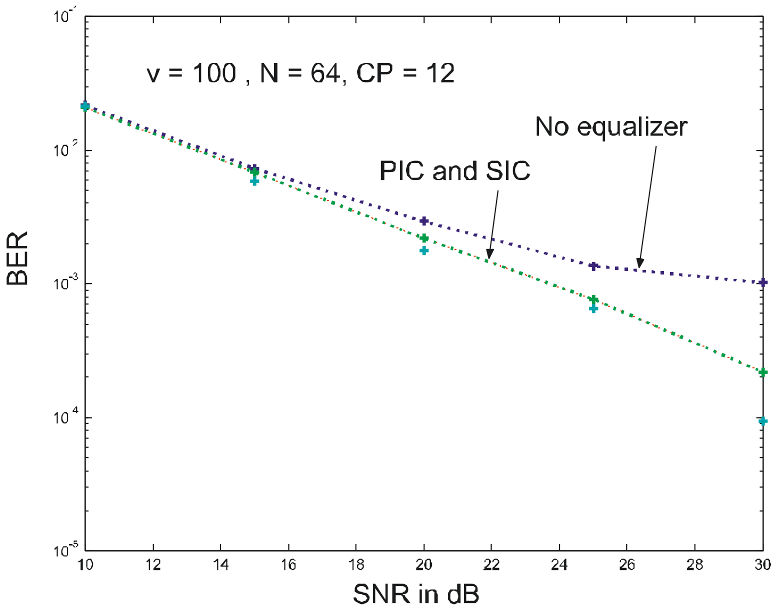
Choose a pulse whose spectrum decays faster which will decrease ICI due to Doppler but faster decay in the frequency domain means slower decay in the time domain which increases delay dispersive errors. Gaussian-shaped basis functions (xmit pulses) result in a good useful compromise

Remedies for ICI (2)

- Self-interference cancellation

  modulate more than one subcarrier for each OFDM symbol but this results in reduced spectral efficiency

- Frequency-domain equalizers requires knowledge of the channel



Don't forget subcarrier synchronization errors and the phase noise associated with RX and TX oscillators inaccuracies

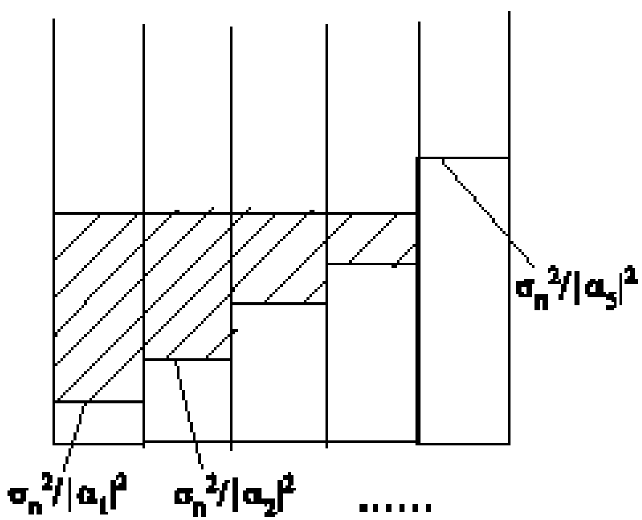
Waterfilling

- To optimize capacity, different powers should be allocated to the subcarriers

Make sure power is not wasted on subcarriers with poor SNR.
For OFDM this means subcarriers in a deep fade

- Waterfilling:

$$P_n = \max\left(0, \varepsilon - \frac{\sigma_n^2}{|\alpha_n|^2}\right) \quad \text{with} \quad P = \sum_{n=1}^{N} P_n$$



no power at all for this channel (subcarrier) with poor SNR

Height proportional to the inverse of the subcarrier's SNR

Section 19.9 describes OFDMA (Orthogonal Frequency Division Multiple Access) which uses OFDM's different subcarriers for different users and not the single user normally associated with OFDM

MUTLICARRIER CDMA (MC-CDMA) AND SINGLE-CARRIER FREQUENCY-DOMAIN EQUALIZATION (SC-FDE)

Note that the OFDM transmitter has available numerous modulation alphabets available for use which can be changed during transmission (BPSK, QPSK, 16-QAM and 64-QAM)

And now for the mathematics…

- A code symbol *c* is mapped onto a transmit vector, by multiplication with spreading code **p**.
- For parallel transmission of symbols: a vector of transmit symbols **c** is mapped by multiplication with a *spreading matrix* **P** that consists of the spreading codes for the different symbols

$$\tilde{\mathbf{c}} = \mathbf{P}\mathbf{c} \qquad \mathbf{P} = \left[\mathbf{p}_1 \; \mathbf{p}_2 \cdots\cdots \mathbf{p}_N\right]$$

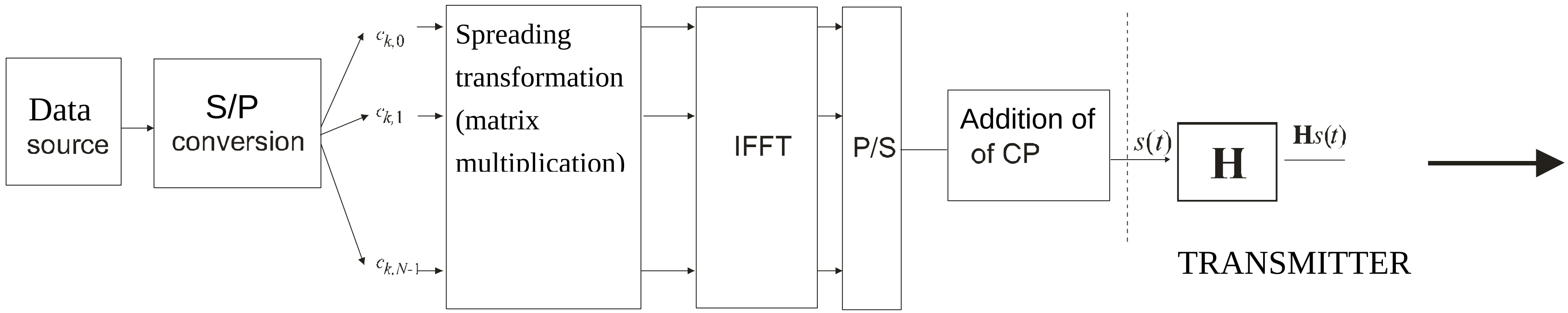
- Symbol spreading is undone at the receiver

$$\tilde{\mathbf{r}} = \mathbf{H}\tilde{\mathbf{c}} + \mathbf{n}$$

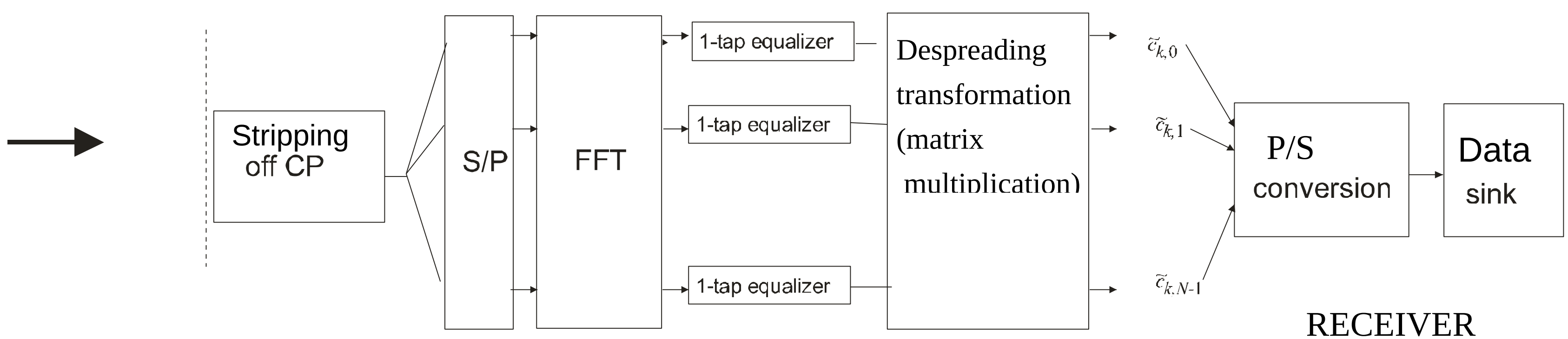
$$\mathbf{P}^{\dagger}\mathbf{H}^{-1}\tilde{\mathbf{r}} = \mathbf{P}^{\dagger}\mathbf{H}^{-1}\mathbf{H}\mathbf{P}\mathbf{c} + \mathbf{P}^{\dagger}\mathbf{H}^{-1}\mathbf{n}$$
$$= \mathbf{c} + \tilde{\mathbf{n}}$$

Basically we're transmitting a data symbol on all available subcarriers simultaneously combining CDMA with OFDM since uncoded OFDM has poor performance

Transceiver structure for MC-CDMA



Multicarrier CDMA is less sensitive to fading with the modulation symbol spread over many tones versus one tone (subcarrier) the the fading we've seen dominates the overall OFDM BER is reduced.

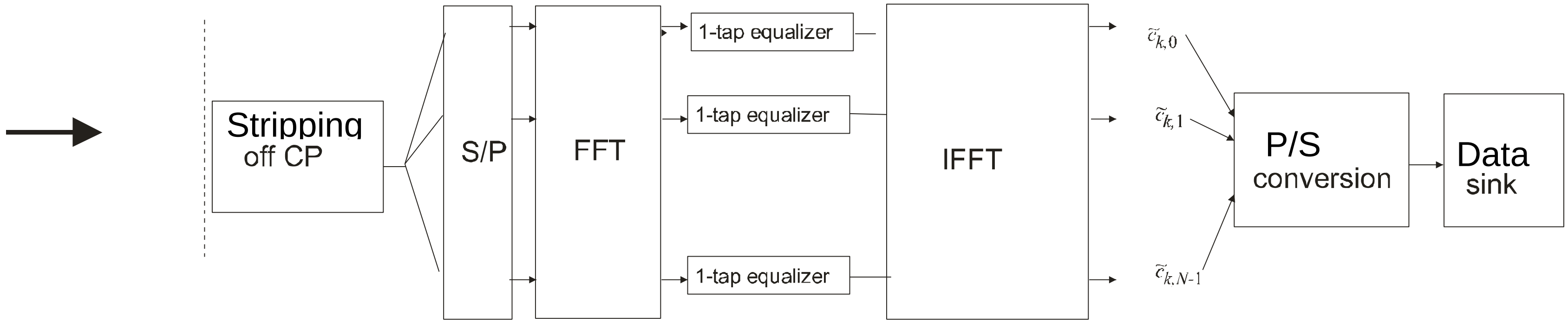


SC-FDE Principle

- Move the IFFT from the TX to the RX

Multiplication by the spreading matrix and the IFFT of OFDM are counter productive to each other but still result in an improvement. The cyclic prefix reduces the impacts of ISI and ICI but with the associated overhead of the cyclic prefix

SC-FDE RECEIVER which performs equalization in the frequency domain. This doesn't provide optimum performance for the equalizer but it is computationaly efficient for the FFT and IFFT.



Single-Carrier Modulation with Frequency Domain Equalization (SC-FDE)