

HW7 DSP Spring 2015 DUE April 15

Problem 1 15 points

- (a) Let $x(t) = \cos(2\pi(t - 4)) + \sin 5\pi t$ and determine the period of each term.
- (b) Determine if $x(t)$ is periodic and determine the fundamental period.
- (c) If $x_1(t) = \cos[2\pi 500 (t - 0.5 * 10^{-3})]$, what is the phase shift in radians for the cos term?

Problem 2

5 Points

Write the function $f(t) = 2 \cos \omega t + 2 \sin \omega t$ as

- (a) $f(t) = C \cos(\omega t + \theta)$ by defining C and θ (in radians).
- (b) As the vector $\vec{x} = a + jb$ defining the magnitude and phase of the function in Part (a) by defining a and b .

Problem 3

10 Points

Show that the $1/2$ power radian frequency of the prototype 2nd order Butterworth filter is $\omega_c = 1$. The transfer function is

$$H(s) = \frac{1}{(s^2 + \sqrt{2}s + 1)}.$$

Problem 4

10 Points

Given the transfer function

$$H(s) = \frac{\omega_c^2}{s^2 + \sqrt{2}\omega_c s + \omega_c^2},$$

show that

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\frac{\omega}{\omega_c})^4}}$$

Problem 5
20 Points

- (a) Compute the a_k coefficients for the Fourier Series of the function

$$f(t) = \begin{cases} -1, & -1 < t < 0, \\ +2, & 0 < t < 2, \end{cases}$$

with $f(t) = f(t + T)$, $T = 3$ for all t since the function is periodic. You might sketch the function to help you.

- (b) Write the trigonometric function of time that goes with your coefficients (2 Points).

Problem 6
20 Points

From the references on the WEB, use the OKAWA low-pass filter design tool to design a Sallen-key low pass filter with the following specifications:

$$f_c = 1000 \text{ Hz}, \zeta = 0.707.$$

Turn in The R_1, R_2, C_1, C_2 values and the circuit diagram of the filter. The factor Q is defined as

$$Q = \frac{1}{2\zeta}.$$

Problem 7
20 Points

1. Design a bandpass filter with bandwidth of 7 kHz centered at 80 kHz. The gain at $\omega = 10^5$ rad/sec should be at least -100 dB.
2. Plot the frequency response between 10^5 and 10^6 rad/sec.

For the filter problems, use the MATLAB command **butter** in the Signal Processing Toolbox. Use also **bode** for the plot.