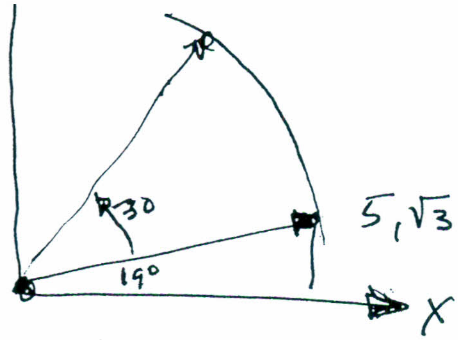
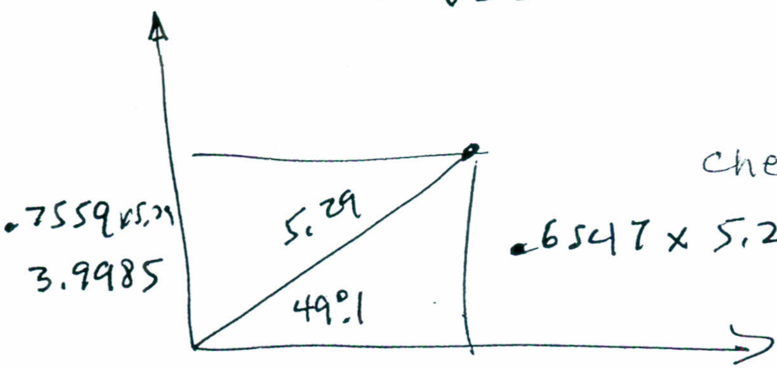
Problem 2



$\sqrt{3}/5 = 0.3464$

$\tan^{-1}(\sqrt{3}/5) = 19.10°$

$L = \sqrt{5^2 + (\sqrt{3})^2}$
$= \sqrt{25+3} = 5.29$



check!

$.6547 \times 5.29 = \underline{3.4636}$

$$\begin{pmatrix} \overset{\sqrt{3}/2}{\cos\theta} & \overset{1/2}{-\sin\theta} \\ \underset{1/2}{\sin\theta} & \underset{\sqrt{3}/2}{\cos\theta} \end{pmatrix} \begin{pmatrix} 5 \\ \sqrt{3} \end{pmatrix} = \begin{pmatrix} 3.464 \\ 4.00 \end{pmatrix}$$

~~cos~~

ANOTHER WAY

$$\vec{z} = (5+\sqrt{3}i)\, e^{i\pi/6} = (5+\sqrt{3}i)(\underset{\sqrt{3}/2}{\cos\tfrac{\pi}{6}} + i\underset{1/2}{\sin\tfrac{\pi}{6}})$$

$$\text{So } \vec{z} = \left(5\tfrac{\sqrt{3}}{2} - \tfrac{\sqrt{3}}{2}\right) + i\left(\tfrac{5}{2} + \sqrt{3}\cdot\tfrac{\sqrt{3}}{2}\right)$$

$$= 2\sqrt{3} + i4 = \underline{3.464 + 4i} \checkmark$$

HW5

4391

PROBLEM 3

KLAFTER p576

$$\vec{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}$$

3a ROTATE 60° about z KLAFTER Eq 8.2.32 EX 8.2.5 p578

$$v_1 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.23 \\ 1.866 \\ 3.0 \\ 1 \end{bmatrix}$$

3 b) ROTATE v_1 by $-90°$ about y axis

KLAFTER EXAMPLE 8.2.6

$$w_1 = \begin{bmatrix} -3 \\ -1.866 \\ -1.2321 \\ 1.0 \end{bmatrix}$$ Klafter Eq 8.2.37

CHECK IT

$$w_1^* = [ROT(y,-90)][ROT(z,60)\,v_1]$$

$$= w_1 \checkmark$$

3c IN OPPOSITE ORDER

$$w = Rot(z,60)\; Rot(y,-90)\; \vec{u}$$

$$\begin{bmatrix} -3.232 \\ -1.598 \\ 1.0 \\ 1.0 \end{bmatrix}$$

MATLAB SOLUTION

```
%------------------------------------------------------------------------
% Problem 2 Rotation of 2D vector
%
% Take the point P=[5,sqrt(3)] and rotate it 30 degrees about origin (z)
P=[5 sqrt(3)]'
Plength=sqrt(P(1)^2+P(2)^2)         % Length is 5.29
Pang=atan(P(2)/P(1))*(180/pi)       % Pang =   19.1066 degrees
%
theta = 30*pi/180                % % Rotation angle theta = 0.5236 rad
Rmat=[cos(theta) -sin(theta);sin(theta) cos(theta)]    % Rotated Point
P1=Rmat*P
P1angle=atan(P1(2)/P1(1))*(180/pi)
%
% P1 =
 %   3.4641
 %   4.0000
% P1angle =    49.1066          % As expected 19.1 + 30 degrees
%-----------------------------------------------------------------------
% Problem 3 3D rotation of homogeneous vector (Klafter P575-580)
%
u=[1 2 3 1]'
theta3=60*pi/180           %theta3 =    1.0472 rad
% 3a Rotation 60 degrees about Z
Rotztheta3=[cos(theta3) -sin(theta3) 0 0;sin(theta3) cos(theta3) 0 0 ;...
    0 0 1 0; 0 0 0 1]
v1=Rotztheta3*u
%
% v1 =              % Check Klafter Eq. 8.2.32 P 578
%   -1.2321
%    1.8660
%    3.0000
%    1.0000
%
%  3b Rotate v1 -90 deg about y

 theta3b=-90*pi/180       % theta3b =   -1.5708 rad
 Rotytheta3b=[ cos(theta3b) 0 sin(theta3b) 0;0 1 0 0;...
     -sin(theta3b) 0 cos(theta3b) 0;0 0 0 1]
 w1=Rotytheta3b*v1
 % w1 =               % Klafter Eq. 8.2.37 P 579
 %   -3.0000
 %    1.8660
 %   -1.2321
 %    1.0000
 %
 % Check result with matrix multiply
w1check=Rotytheta3b*(Rotztheta3*u)    %Note Order RZ60 THEN RY-90 It Checks
%
% 3c  Rotate in opposite order RY-90 THEN RZ60
w=Rotztheta3*(Rotytheta3b*u)    % Matrices DO NOT COMMUTE !Klafter Eq 8.2.39
% w =               % A different end point
%   -3.2321
%   -1.5981
%    1.0000
%    1.0000
%
```

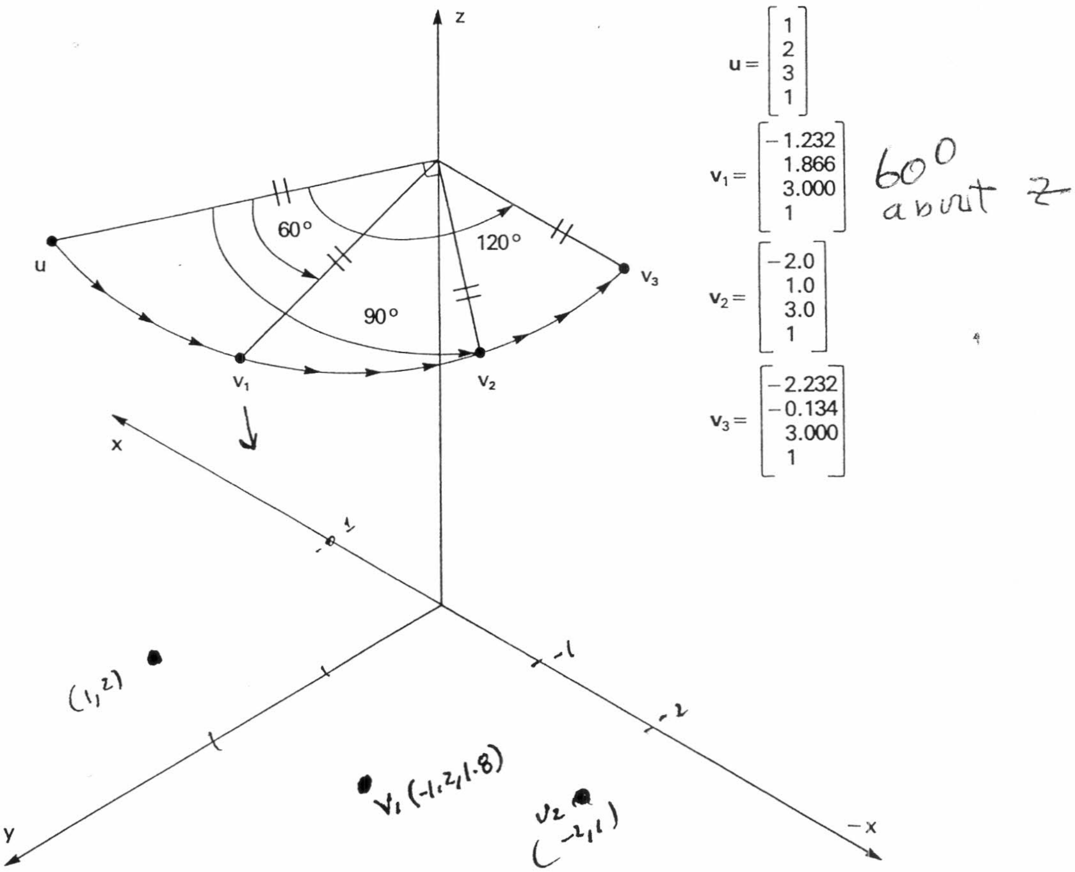


Figure 8.2.6. Rotation of a point about the z axis.

at a height of 3 units above the x-y reference plane which contains the point (1, 2). The relationship of the point (1, 2) remains the same to the reference frame of the plane in which it is located but is different from the base or original x-y reference frame located at height $z = 0$. Using Eq. (8.2.27) we will consider rotations of 60°, 90°, and 120°. The operators corresponding to these angles are given as

$$\text{Rot}\,(z, 60) = \begin{bmatrix} 0.500 & -0.866 & 0 & 0 \\ 0.866 & 0.500 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8.2.29}$$

$$\text{Rot}(z, 90) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8.2.30}$$

Since matrix operations are generally not commutative, the order in which the rotations are performed becomes quite important. In general, the product of elementary rotational transformations taken in different order will yield different results. Consider first a rotation of $-90°$ about the y axis followed by a rotation of $60°$ about the z axis. The product of the two elementary transformations is given by

$$\mathrm{Rot}(z, 60)\ \mathrm{Rot}(y, -90) = \begin{bmatrix} 0 & -0.866 & -0.500 & 0 \\ 0 & 0.500 & -0.866 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8.2.38}$$

The result of this transformation operating on $\mathbf{u}$ is shown in Figure 8.2.8. The new vector becomes

$$\mathbf{w}' = \begin{bmatrix} -3.232 \\ -1.598 \\ 1.0 \\ 1.0 \end{bmatrix} \tag{8.2.39}$$

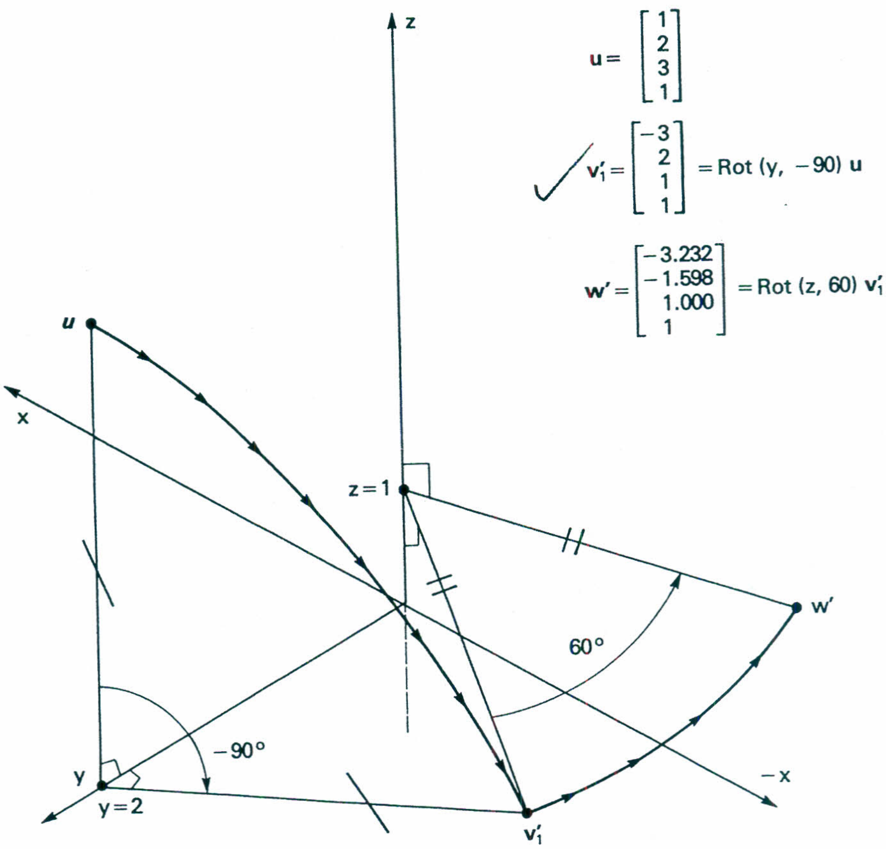


Figure 8.2.8. Rot (z, 60°) Rot (y, −90°).